IRSTI 29.01.11 https://doi.org/10.26577/JPEOS.2023.v25.i3-4.i1

S.H. Sohrab

Northwestern University, U.S.A., Illinois, Evanston
e-mail: s-sohrab@northwestern.edu

INVARIANT MODEL OF BOLTZMANN STATISTICAL MECHANICS AND ITS IMPLICATIONS TO HYDRODYNAMIC MODEL OF ELECTROMAGNETISM, PHYSICAL FOUNDATIONS OF QUANTUM MECHANICS, RELATIVITY, QUANTUM GRAVITY AND QUANTUM COSMOLOGY

Some implications of an invariant model of Boltzmann statistical mechanics to quantum nature of (space, time, mass), physical foundations of quantum mechanics, relativity, electromagnetism and Maxwell equations, quantum gravity and quantum cosmology will be discussed. In harmony with Huygens' analogy between propagation of sound in air and light in ether, propagation of light wave is shown to involve an exceedingly thin longitudinal component besides its Maxwellian transverse polarizations. Following Maxwell and Lorentz, an invariant hydrodynamic model of electromagnetism is presented. It is expected that time-dependence of the speed of light, identified as root-mean-square velocity of gravitons, could be determined by measurement of changes in period of geological events caused by periodic cosmic radiation bursts from sources such as pulsars. New interpretation of physical foundation of quantum mechanics, Dirac wave equation, pilot waves of de Broglie-Bohm model will be described.

Identifying physical space, Aristotle fifth element or Casimir vacuum, as a tachyonic compressible fluid, in harmony with Huygens and Planck compressible ether, Lorentz-FitzGerald contractions becomes causal (Pauli) leading to Poincaŕe-Lorentz dynamic as opposed to Einstein kinematic theory of relativity. Invariant forms of conservation equations lead to hydrodynamics of universe governed by quantum gravity as a dissipative deterministic dynamic system proposed by 't Hooft. The thermodynamics of universe suggests possible relevance of classical Nordström scalar, and Abraham vector theories of gravitation beside Einstein tensor theory. Also, some implications of the model to quantum cosmology, loop quantum gravity (LQG), and Everett multiverse are discussed.

Key words: electromagnetism, Maxwell equations, quantum mechanics, quantum cosmology, quantum gravity, relativity, T.O.E.

С.Х. Сохраб

Норствестерн университеті, АҚШ, Иллинойс штаты, Эванстон қ.
e-mail: s-sohrab@northwestern.edu

Больцманның статистикалық механикасының инвариантты моделі және оның электромагнетизмнің гидродинамикалық моделіне, кванттық механиканың физикалық негіздеріне, салыстырмалылық, кванттық гравитацияға және кванттық космологияға әсері

Мақалада Больцман статистикалық механикасының инвариантты моделінің кванттық табиғаты (кеңістік, уақыт, масса), кванттық механиканың физикалық негіздері, салыстырмалылық теориясы, электромагнетизм және Максвелл теңдеулері, кванттық гравитация және кванттық космологияның кейбір салдары талқыланады. Гюйгенстің ауадағы дыбыс пен жарықтың таралуы арасындағы ұқсастығына сәйкес, жарық толқынының таралуы көлденең поляризациядан басқа өте жұқа бойлық құрамдас бөлікті қамтиды. Максвелл мен Лоренцтен бөлек электромагнетизмнің инвариантты гидродинамикалық моделі ұсынылған. Гравитондардың орташа квадраттық жылдамдығы ретінде анықталған жарық жылдамдығының уақытқа тәуелділігін пульсарлар сияқты көздерден ғарыштық радиацияның жарқылынан туындаған геологиялық оқиғалар кезеңіндегі өзгерістерді өлшеу арқылы анықтауға болады деп күтілуде. Кванттық механиканың физикалық негіздерінің жаңа түсіндірмесі, Дирак толқын теңдеуі, де Бройль-Бом моделінің пилоттық толқындары сипатталады.

Физикалық кеңістікті, Аристотельдің бесінші элементін немесе Касимир вакуумын, Гюйгенс пен Планктың сығылатын эфирімен үйлесімде тахиондық сығылатын сұйықтық ретінде, Лоренц-Фиц-Джеральд жиырылулары Эйнштейннің кинематикалық реактивтілікке қарсы Пуанкаре-Лоренц динамикасына әкелетін себептік (Паули) болады. Сақтау теңдеулерінің инвариантты формалары Т Хоофт ұсынған диссипативті детерминирленген динамикалық жүйе ретінде кванттық гравитациямен басқарылатын ғаламның гидродинамикасына әкеледі. Әлемнің термодинамикасы Эйнштейннің тензорлық теориясымен қатар классикалық Нордстрем скалярының және Авраамның векторлық гравитация теорияларының ықтимал сәйкестігін болжайды. Сондай-ақ, модельдің кванттық космология, циклдік кванттық гравитация және Эвереттің көптік әлеміне қатысты кейбір салдар талқыланады.

Түйін сөздер: электромагнетизм, Максвелл теңдеулері, кванттық механика, кванттық космология, кванттық гравитация, салыстырмалылық, Т.О.Е.

С.Х. Сохраб

Университ Норсвестерн, штат Иллинойс, США, г. Еванстон
e-mail: s-sohrab@northwestern.edu

Инвариантная модель статистической механики Больцмана и ее последствия для гидродинамической модели электромагнетизма, физических основ квантовой механики, теории относительности, квантовой гравитации и квантовой космологии

В статье будут обсуждаться некоторые последствия инвариантной модели статистической механики Больцмана для квантовой природы (пространства, времени, массы), физических основ квантовой механики, теории относительности, электромагнетизма и уравнений Максвелла, квантовой гравитации и квантовой космологии. В соответствии с аналогией Гюйгенса между распространением звука в воздухе и света в эфире показано, что распространение световой волны включает в себя чрезвычайно тонкую продольную составляющую, помимо максвелловской поперечной поляризации. Вслед за Максвеллом и Лоренцем представлена инвариантная гидродинамическая модель электромагнетизма. Ожидается, что временная зависимость скорости света, определяемая как среднеквадратическая скорость гравитонов, может быть определена путем измерения изменений периода геологических событий, вызванных периодическими всплесками космического излучения от таких источников, как пульсары. Будет описана новая интерпретация физических основ квантовой механики, волнового уравнения Дирака, пилотных волн модели де Бройля-Бома.

Идентифицируя физическое пространство, пятый элемент Аристотеля или вакуум Казимира, как тахионную сжимаемую жидкость, в гармонии со сжимаемым эфиром Гюйгенса и Планка, сокращения Лоренца-Фитцджеральда становятся причинными (Паули), что приводит к динамике Пуанкаре-Лоренца, в отличие от кинематической теории относительности Эйнштейна. Инвариантные формы уравнений сохранения приводят к гидродинамике Вселенной, управляемой квантовой гравитацией как диссипативной детерминированной динамической системой, предложенной Т Хоофтом. Термодинамика Вселенной предполагает возможную значимость классической скалярной теории Нордстрема и векторной теории Абрагама гравитации рядом с тензорной теорией Эйнштейна. Также обсуждаются некоторые последствия модели для квантовой космологии, петлевой квантовой гравитации и мультивселенной Эверетта.

Ключевые слова: электромагнетизм, уравнения Максвелла, квантовая механика, квантовая космология, квантовая гравитация, относительность. Т.О.Е.

Introduction

What is common amongst the diverse fields of chromodynamics, electrodynamics, turbulent hydrodynamics, astrophysics, and cosmology is that they all involve statistical fields composed of very large number of "particles" and particle clusters under turbulent motion. Guided by similarities between small scale stochastic quantum fields [1-17] and large scale turbulent hydrodynamic fields [18-30], a scale invariant model of Boltzmann statistical mechanics was recently developed and applied to the investigation of fluid mechanics [31, 32], thermodynamics [33, 34], and quantum mechanics [35, 36, 37] at large, intermediate, and small scales.

The present study is focused on the application of invariant model of Boltzmann statistical mechanics to: (I) invariant model of quantization of space, time, and mass (II) physical foundation of quantum mechanics and theory of relativity (III) physical foundation of electromagnetism and invariant Maxwell equations (IV) physical foundations of quantum gravity and quantum cosmology. First, a very brief description of invariant model of Boltzmann statistical mechanics and conservation equations are discussed. Next, quantization of space, time, and mass and their invariant definitions are studied. Internal spacetime versus external space and time are described and the latter is identified as Rovelli *thermal time*. Next, a hydrodynamic model of Maxwell theory of electromagnetism is presented. Finally, some implications of invariant Schrödinger equation to cosmology, quantum gravity, quantum cosmology, and multiverse are discussed.

Scale–Invariant Model of Boltzmann Statistical Mechanics and Invariant Conservation Equations

The scale-invariant model of statistical mechanics for equilibrium galactic-, planetary-, hydro-system-, fluid-element-, eddy-, cluster-, molecular-, atomic-, subatomic-, kromo-, and tachyon-dynamics corresponding to the scale β = g, p, h, f, e, c, m, a, s, k, and t is schematically shown on the left hand side of Fig. 1.

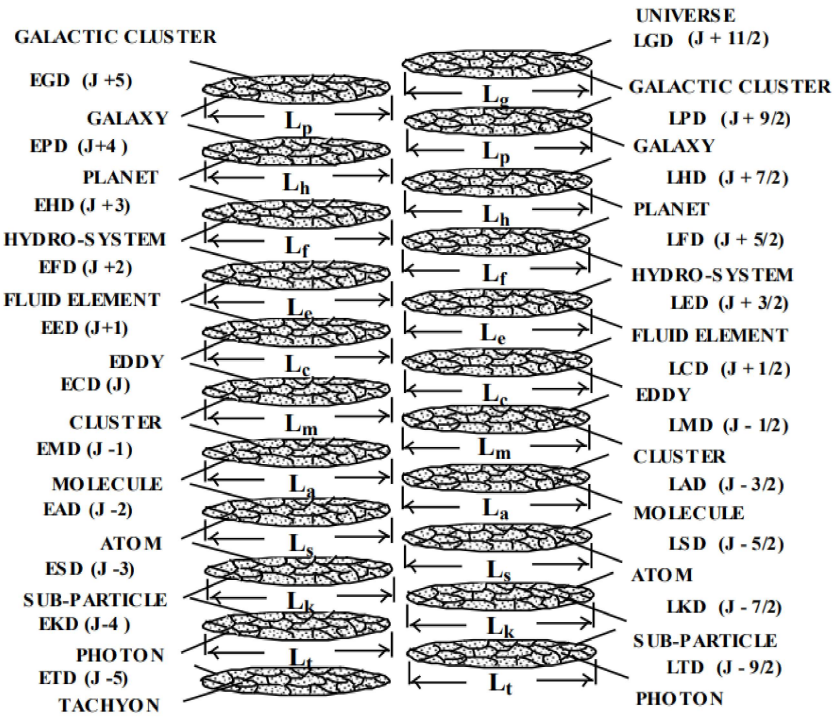


Figure 1 – A scale-invariant model of statistical mechanics. Equilibrium-β-Dynamics on the left-hand-side and non-equilibrium Laminar-β-Dynamics on the right-hand-side for scales β = g, p, h, f, e, c, m, a, s, k, and t as defined in [36]. Characteristic lengths of (system, element, "atom") are $(L_\beta, \lambda_\beta, \ell_\beta)$ and λ_β is the mean-free-path.

For each statistical field, one defines particles that form the background fluid and are viewed as point-mass or "*atom*" of the field. Next, the *elements* of the field are defined as finite-sized composite entities composed of an ensemble of "*atoms*". Finally, ensemble of a large number of "*elements*" is defined as the statistical "*system*" at that particular scale. The most-probable element of scale β defines the "atom" (system) of the next higher β+1 (lower β–1) scale.

Following the classical methods [19, 38-42], the invariant definitions of the density ρ_β, and the velocity of *atom* $\mathbf{u}_\beta$, *element* $\mathbf{v}_\beta$, and *system* $\mathbf{w}_\beta$ at the scale β are given as [31,36]

$$\rho_\beta = n_\beta m_\beta = m_\beta \int f_\beta du_\beta, \quad \mathbf{u}_\beta = \mathbf{v}_{w\beta-1} \tag{1}$$

$$\mathbf{v}_\beta = \rho_\beta^{-1} m_\beta \int \mathbf{u}_\beta f_\beta du_\beta, \quad \mathbf{w}_\beta = \mathbf{v}_{w\beta+1} \tag{2}$$

Similarly, the invariant definitions of the peculiar and diffusion velocities are introduced as

$$\mathbf{V}'_\beta = \mathbf{u}_\beta - \mathbf{v}_\beta, \quad \mathbf{V}_\beta = \mathbf{v}_\beta - \mathbf{w}_\beta, \quad \mathbf{V}_\beta = \mathbf{V}'_{\beta+1} \tag{2}$$

Following the classical methods [19, 38-42], the scale-invariant forms of mass, thermal energy, linear and angular momentum conservation equations at scale β are given as [31, 36]

$$\frac{\partial \rho_{i\beta}}{\partial t_\beta} + \nabla.(\rho_{i\beta}\mathbf{v}_\beta) = \Re_{i\beta} \tag{4}$$

$$\frac{\partial \varepsilon_{i\beta}}{\partial t_\beta} + \nabla.(\varepsilon_{i\beta}\mathbf{v}_\beta) = 0 \tag{5}$$

$$\frac{\partial \mathbf{p}_{i\beta}}{\partial t_\beta} + \nabla.(\mathbf{p}_{i\beta}\mathbf{v}_\beta) = -\nabla.\mathbf{P}_{ij\beta} \tag{6}$$

$$\frac{\partial \boldsymbol{\pi}_{i\beta}}{\partial t_\beta} + \nabla.(\boldsymbol{\pi}_{i\beta}\mathbf{v}_\beta) = \rho_{i\beta}\boldsymbol{\omega}_\beta.\nabla\mathbf{v}_\beta \tag{7}$$

involving the *volumetric density* of thermal energy $\varepsilon_{i\beta} = \rho_{i\beta}\tilde{h}_{i\beta}$, linear momentum $\mathbf{p}_{i\beta} = \rho_{i\beta}\mathbf{v}_{i\beta}$, and angular momentum $\boldsymbol{\pi}_{i\beta} = \rho_{i\beta}\boldsymbol{\omega}_{i\beta}$ (since $r_{a\beta-1} = 1$). Also, $\Re_{i\beta}$ is the chemical reaction rate and $\tilde{h}_{i\beta} = \hat{h}_{i\beta} / m_\beta$ is the absolute enthalpy [36]. It is noted that the time coordinates in equations (4-7) also have a scale subscript β.

As discussed in an earlier study on classical thermodynamics [34], and in harmony with perceptions of Clausius [43, 44], atoms of the field are assumed to have three *simultaneously independent* degrees of freedom associated with their translational, rotational, and vibrational (pulsation) motions. Thus, in axisymmetric cylindrical coordinates (z, θ, r), particles are assumed to undergo internal harmonic translation, rotation, and pulsation in two axial (z+, z–), angular (θ+, θ–), and radial (r+, r-) directions. The classical definition of vorticity involves the curl of linear velocity $\nabla \times \mathbf{v}_\beta = \boldsymbol{\omega}_\beta$ thus giving rotational velocity a secondary status since it depends on particle translational velocity $\mathbf{v}_\beta$. However, it is known that particle's rotation about its center of mass is independent of the translational motion of its center of mass. In other words, translational, rotational, and vibrational (pulsation) motions of particle are independent degrees of freedom that should not be necessarily coupled. To resolve this paradox, iso-spin of particle at scale β is defined as the curl of the velocity at the next lower scale of β – 1 [36]

$$\boldsymbol{\varpi}_\beta = \nabla \times \mathbf{u}_\beta = \nabla \times \mathbf{v}_{\beta-1} = \boldsymbol{\omega}_{\beta-1} \tag{8}$$

such that rotational velocity, while having a connection to some type of translational motion at internal scale β – 1 retains its independent degree of freedom at the external scale β as desired. A schematic description of iso-spin and vorticity fields is shown in Fig. 2.

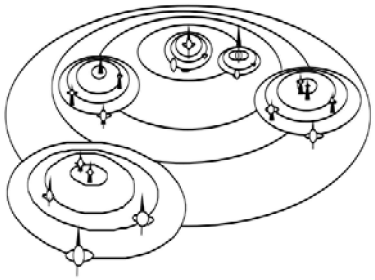


Figure 2 – Description of internal (iso-spin) versus external vorticity fields in cosmology [36].

The nature of galactic vortices in cosmology and the associated dissipation have been discussed [26, 45].

The invariant continuity equation (4) involves invariant form of classical formula for "chemical reaction rate" defined as [40]

$$\Re_{i\beta} = m_{i\beta}\int(\delta f_{i\beta}/\delta t)du_{i\beta} \tag{9}$$

To reveal the scale invariant form of the reaction rate $\Re_g$, it is noted that

$$\sum_i \Re_{i\beta} = \sum_i m_{i\beta}\int(\delta f_{i\beta}/\delta t)du_{i\beta} =$$

$$= \frac{\delta}{\delta t}\sum_i m_{i\beta}\int f_{i\beta}du_{i\beta} = \frac{\delta}{\delta t}\sum_i m_{i\beta}n_{i\beta} =$$

$$= \frac{\delta}{\delta t}\sum_i \rho_{i\beta} = \frac{\delta\rho_{\beta+1}}{\delta t} = \frac{\delta}{\delta t}(m_{\beta+1}n_{\beta+1}) =$$

$$= m_{\beta+1}\int(\delta f_{\beta+1}/\delta t)du_{\beta+1} = \Re_{\beta+1} \tag{10}$$

The local velocity $\mathbf{v}_\beta$ in (4)-(7) is expressed in terms of convective $\mathbf{w}_\beta$ and diffusive $\mathbf{V}_\beta$ velocities [36]

$$\mathbf{w}_\beta = \mathbf{v}_\beta - \mathbf{V}_{\beta g},\ \mathbf{V}_{\beta g} = -D_\beta\nabla\ln\rho_\beta \tag{11}$$

$$\mathbf{w}_\beta = \mathbf{v}_\beta - \mathbf{V}_{\beta tg}.\ \mathbf{V}_{\beta tg} = -\alpha_\beta\nabla\ln\varepsilon_\beta \tag{12}$$

$$\mathbf{w}_\beta = \mathbf{v}_\beta - \mathbf{V}_{\beta hg}.\ \mathbf{V}_{\beta hg} = -\tilde{\kappa}_\beta\nabla\ln\mathbf{p}_\beta \tag{13}$$

$$\mathbf{w}_\beta = \mathbf{v}_\beta - \mathbf{V}_{\beta rhg}.\ \mathbf{V}_{\beta rhg} = -\tilde{\kappa}_\beta\nabla\ln\boldsymbol{\pi}_\beta \tag{14}$$

where $(\mathbf{V}_{\beta g}, \mathbf{V}_{\beta tg}\mathbf{V}_{\beta hg}, \mathbf{V}_{\beta rhg})$ are respectively diffusive, thermo-diffusive, translational and rotational hydro-diffusive velocities and $\tilde{\kappa}_\beta = \eta_\beta/\rho_\beta$ is kinematic viscosity.

Because by definition fluids can only support compressive normal forces, following *Cauchy*, the total stress tensor for fluids is expressed as [46, 47]

$$P_{ij\beta} = p_\beta\delta_{ij} + (\lambda_\beta + \frac{1}{3}\eta_\beta)\nabla.\mathbf{v}_\beta\delta_{ij} \tag{15}$$

Also, the classical Stokes assumption of zero bulk viscosity relates the two *Lame* constants of ideal fluid by [47]

$$b_\beta = \lambda_\beta + \frac{2}{3}\eta_\beta = 0 \tag{16}$$

Hence, the total stress tensor (15) becomes [36]

$$P_{ij\beta} = p_\beta\delta_{ij} - \frac{1}{3}\eta_\beta\nabla.\mathbf{v}_\beta\delta_{ij} = (p_{t\beta} + p_{h\beta})\delta_{ij} \tag{17}$$

where the hydrodynamic pressure is

$$p_{h\beta} = (\tau_{xx\beta} + \tau_{yy\beta} + \tau_{zz\beta})/3 = -\frac{1}{3}\eta_\beta\nabla.\mathbf{v}_\beta \tag{18}$$

The occurrence of a single rather than two Lame constants is in accordance with the perceptions of Cauchy and Poisson who both assumed the limit of zero for the expression [48]

$$\lambda'_\beta + \eta_\beta = \lim_{R\to 0} R^4 f(R) \tag{19}$$

Therefore, in *Cauchy-Poisson* limit (19), as intermolecular spacing vanishes $R \to 0$, all tangential stresses will vanish as was emphasized by *Darrigol* [48]

"Poisson and Cauchy both assumed the limit to be zero. Then the medium loses its rigidity since the transverse pressures disappear"

leaving only normal stresses in (17).

Following the classical methods [19, 38-42], by substituting from (11)-(16) into (4)-(7), neglecting cross-diffusion terms, and applying Onsager [49] reciprocity relations, the invariant forms of conservation equations are written as [36]

$$\frac{\partial\rho_{i\beta}}{\partial t_\beta} + \mathbf{w}_\beta.\nabla\rho_{i\beta} = D_{i\beta}\nabla^2\rho_{i\beta} + \Re_{i\beta} \tag{20}$$

$$\frac{\partial T_{i\beta}}{\partial t_\beta} + \mathbf{w}_\beta.\nabla T_{i\beta} = \alpha_{i\beta}\nabla^2 T_{i\beta} + \tilde{h}_{i\beta}\Re_{i\beta}/\rho_{i\beta}c_{p\beta} \tag{21}$$

$$\frac{\partial \mathbf{v}_\beta}{\partial t_\beta} + \mathbf{w}_\beta.\nabla\mathbf{v}_\beta = \tilde{\kappa}_\beta\nabla^2\mathbf{v}_\beta - \frac{\nabla p_\beta}{\rho_\beta} + \frac{1}{3}\tilde{\kappa}_\beta\nabla(\nabla.\mathbf{v}_\beta) - \frac{\mathbf{v}_\beta\Re_\beta}{\rho_\beta} \quad (22)$$

$$\frac{\partial \boldsymbol{\omega}_{i\beta}}{\partial t_\beta} + \mathbf{w}_\beta.\nabla\boldsymbol{\omega}_{i\beta} = \tilde{\kappa}_\beta\nabla^2\boldsymbol{\omega}_{i\beta} + \boldsymbol{\omega}_{i\beta}\nabla.\mathbf{v}_{i\beta} - \frac{\boldsymbol{\omega}_{i\beta}\Re_{i\beta}}{\rho_{i\beta}} \quad (23)$$

Modified form of the viscous equation of motion (22) is to be compared with the classical Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}.\nabla\mathbf{v} = \tilde{\kappa}\nabla^2\mathbf{v} - \frac{\nabla p}{\rho} + \frac{1}{3}\tilde{\kappa}\nabla(\nabla.\mathbf{v}) \quad (24)$$

The history of derivation of (24) has been studied in an excellent investigation by Darrigol [46]. The main difference between equations (22) and (24) is the occurrence of the global (not differentiable) convective velocity $\mathbf{w}_\beta$ as opposed to local (differentiable) velocity $\mathbf{v}_\beta$ in the second term. Thus, the former equation is more in harmony with the pioneering studies by Oseen [50] and Carrier [51]. As a result, in the absence of convection, (22) reduces to nonhomogeneous diffusion equation similar to (20) and (21). However, the absence of local velocity results in vanishing of almost the entire classical equation of motion (24). In Section 6, solution of the modified form of equation of motion (22) for the classical problem of Stokes viscous flow across a rigid cylinder and resolution of Stokes paradox will be discussed.

Similarly, modified form of Helmholtz [52] vorticity equation (23) is to be compared with the classical equation in absence of chemical reaction

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \mathbf{v}.\nabla\boldsymbol{\omega} = \tilde{\kappa}\nabla^2\boldsymbol{\omega} + \boldsymbol{\omega}\nabla.\mathbf{v} \quad (25)$$

An important difference between the modified (23) and the classical (25) forms of *Helmholtz* vorticity equation is the occurrence of convective velocity $\mathbf{w}_\beta$ as opposed to local velocity $\mathbf{v}_\beta$ in the second term. Because local vorticity $\boldsymbol{\omega}_\beta$ in (25) is itself related to the curl of local velocity it cannot be advected by this same velocity. On the other hand, advection of local vorticity by convective velocity $\mathbf{w}_\beta$ in (23) is possible. Moreover, in absence of convection (23) reduces to the diffusion equation similar to that in (20), (21), and (22) for mass, heat, and linear momentum transport. However, the absence of local velocity in (25) leads to vanishing of the entire equation.

Hierarchies of Quantum Fields with Atomic Mass Unit, Internal Spacetime, and External Space and Time

Like all statistical fields in the hierarchy shown in Fig. 1, the field of stochastic chromodynamics that is the physical space is also real. It is well known that in late 1800's, most scientists believed that space was filled with a fluid called luminiferous ether that was the seat of electromagnetic waves. In recent investigations [35-37], the ether was identified as the physical space itself rather than a substance filling the space. To use a glass of water as an example, physical space is identified as water itself rather than the glass. Hence, physical space is identified as a compressible tachyon fluid in harmony with perceptions of Aristotle [53], Huygens [54], Newton [55], Euler [56], Maxwell [57], Lorentz [58], Poincaré [59-62], de Broglie [3], Casimir [63], and Dirac [64] amongst many others. The history of classical ether theories is discussed in an excellent book by Whittaker [65].

As discussed in [36], physical space or Casimir vacuum [63] is identified as a compressible tachyonic fluid schematically shown in Fig. 3, from infinite rarefaction (*white hole* WH) to infinite compression (*black hole* BH).

Hence, four significant characteristics of ether are (a) ether is the physical space or Casimir [63] vacuum itself and not its occupier (b) in accordance with perceptions of Huygens [54], ether is a compressible fluid (c) ether is *stochastically stationary* because, as emphasized by Dirac [64], static ether cannot satisfy both relativity theory and quantum mechanics (d) ether is an atomic thus quantum fluid (Fig. 1).

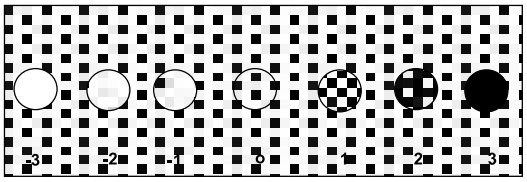


Figure 3 – Different degrees of rarefaction and compression of Casimir vacuum identified as a compressible fluid. **(-3)** White hole $\rho_{WH} = 0$ **(-2, -1)** Anti-matter $\rho_{AM} < \rho_v$ **(0)** Casimir vacuum $\rho = \rho_v$ **(1, 2)** Matter $\rho_{MA} > \rho_v$ **(3)** Black hole $\rho_{BH} = \infty$ [37].

Among various properties of physical space, its compressibility plays the most central role in physical foundations of theory of relativity discussed in next Section, and propagation of light in hydrodynamic model of Maxwell [57] theory of electromagnetism discussed in Section 5. Therefore, we begin the discussion of space and time with an important quotation from Huygens' treatise on light concerning analogy between propagation of sound in air and that of light in ether [54]

"As regards the different modes in which I have said the movement of Sound and of Light are communicated, one may sufficiently comprehend how this occurs in the case of Sound if one considers that the air is of such nature that it can be compressed and reduced to a much smaller space than that which it ordinarily occupies,"

"Now in applying this kind of movement to that which produces Light there is nothing to hinder us from estimating the particles of the ether to be of a substance as nearly approaching to perfect hardness and processing a springiness as prompt as we choose. It is not necessary to examine here the causes of this hardness, or of that springiness, the consideration of which would lead us too far from our subject. I will say, however, in passing that we may conceive that the particles of the ether, notwithstanding their smallness, are in turn composed of the other parts and that their springiness consists in the very rapid movement of a subtle matter which penetrates them from every side and constrains their structure to assume such disposition as to give to this fluid matter the most overt and easy passage possible. This accords with the explanation which Mr. Des Cartes gives for the spring, though I do not, like him, suppose the pores to be in the form of round hollow canals. And it must not be thought that in this there is anything absurd or impossible, it being on the contrary quite credible that it is this infinite series of different sizes of corpuscles, having different degrees of velocity, of which Nature makes use to produce so many marvelous effects,"

Therefore, Huygens considered atoms of ether to be further divisible and have different velocities in harmony with infinite hierarchies of statistical fields shown in Fig. 1. This is also in harmony with perceptions of Lorentz [66] who believed that ether fluid penetrates electrons as atoms of electrodynamics field

"Now, if within an electron there is ether, there can also be an electromagnetic field, and all we have got to do is to establish a system of equations that may be applied as well to the parts of the ether where there is an electric charge, i.e. to the electrons, as to those where there is none."

Also, the significance and central role of ether as carrier of electromagnetic waves in theory of electron was emphasized by Lorentz [58]

"I cannot but regard the ether, which is the seat of an electromagnetic field with its energy and its vibrations, as endowed with certain degree of substantiality, however different it may be from all ordinary matter"

Finally, even though he believed in relational concepts of space and time, the importance of reality of absolute space was emphasized by Leibniz [67]

"If space is an absolute reality, far from being a property or accident opposed to substance, it will have a greater reality than substances themselves,"

When physical space is identified as a compressible fluid described by Boltzmann statistical mechanics at thermodynamic equilibrium, the two *universal constants* of nature namely Planck and Boltzmann constants (h, k) are related to stochastic *spatial* and *temporal* aspects of Casimir [63] vacuum fluctuations as [34]

$$h = h_k = m_k c < \lambda_k^2 >^{1/2} = 6.626 \times 10^{-34} \text{ J-s} \quad (26)$$

$$k = k_k = m_k c < \nu_k^2 >^{1/2} = 1.381 \times 10^{-23} \ \text{J/m} \quad (27)$$

Stochastically stationary Planck and Boltzmann constants in (26-27) lead to four invariant *universal* thermodynamic properties namely gravitational mass of photon, atomic-mass-unit *amu*, Avogadro-Loschmidt number, and universal gas constant [34]

$$m_k = (hk / c^3)^{1/2} \approx 1.84278 \times 10^{-41} \ \text{kg} \quad (28)$$

$$amu = m_k c^2 = (hkc)^{1/2} \approx 1.6563 \times 10^{-27} \ \text{kg} \quad (29)$$

$$N^o = 1/(m_k c^2) = 1/(hkc)^{1/2} \approx 6.0376 \times 10^{26} \ \text{kmol}^{-1} \quad (30)$$

$$R^o = N^o k = (k / hc)^{1/2} \approx 8.338 \ \text{kJ/kmol-m} \quad (31)$$

According to (29), all atoms are composed of photons such that at the most fundamental level, the entire universe is constructed from physical space i.e., Casinir [63] vacuum in harmony with perceptions of Leibniz described in quotation above. Possible electromagnetic nature of all matter was anticipated by Newton [55], Lorentz [58], and Poincaré [59].

It was emphasized by Pauli [68] that

"No quantity with the dimension of time occurs in thermodynamics; at the most, time enters as the concept of "before" and "after". Therefore, in the case of rapidly occurring processes, only initial and final states are discussed."

More recently, Rovelli [69] in an excellent book about nature of time, compares Aristotle's concept of time as a measure of change with Newton concept of absolute time. Also, time was suggested to have a thermodynamic origin by Rovelli [70,71]

"The theory seems to indicate that there is no explanation for the peculiar properties of the time variable at the mechanical level. We suggest that such an explanation should be searched at the thermodynamical level. We propose the idea that the very concept of time is meaningful only in the thermodynamic context"

The nature of thermodynamic, psychological, as well as cosmological arrows of time were discussed by Hawking [72],

"Both Aristotle and Newton believed in absolute time. That is, they believed that one could unambiguously measure the interval of time between two events, and this time would be the same whoever measured it, provided they used a good clock. Time was completely separate from and independent of space. This is what most people would take to be the commonsense view. However, we have had to change our ideas about space and time. Although our apparently commonsense notions work well when dealing with apples, or planets that travel comparatively slowly, they don't work at all for things moving at or near the speed of light."

Concerning Aristotle's notion of time, we quote from Vladimirov et. al., [73]

"In the same manner that he denied the existence of the vacuum, Aristotle refused to accept the notion that time is independent of events. His argument was that time cannot exist without change, that is if the present were not different in each situation but remained the same, there would be no time,"

Therefore, perhaps from a different perspective, it may be said that Aristotle did not believe in the notion of *absolute time* contrary to assertion in the above quotation from Hawking [72]. In other words, if time is to be defined in terms of the state of *change* which is itself known to be easily altered, the notion of absoluteness may no longer be attributed to such a time. It is most natural to maintain that in the absence of all motions there is no time at all, since the concept of time in the *absence of all motions* (changes) is an empty and meaningless concept as emphasized by Aristotle [53].

Although the dimension of time does not explicitly occur in thermodynamics according to above quotation from Pauli [68], both space and time do occur in the definition of thermodynamic temperature as it relates to the kinetic energy of oscillators [74]

$$m_\beta u_\beta^2 = m_{\beta-1} v_{w\beta-1}^2 = kT_{\beta-1} = k\lambda_{w\beta-1} \quad (32)$$

resulting in introduction of the concepts of *internal spacetime* versus *external space and time* [74]. Hence, by definition of most-probable or Wien speed $v_{w\beta} = \lambda_{w\beta} \nu_{w\beta} = \lambda_{w\beta} / \tau_{w\beta}$, one associates

constant internal measures of (extension, duration) [74]

$$\begin{cases} \lambda_{ws} & \text{Internal measure of extension} \\ \tau_{ws} & \text{Internal measure of duration} \end{cases} \quad (33)$$

with every "point" or "atom" of space at constant temperature $T_\beta = T_{\beta-1}$.

According to Fig.1, the statistical field $\mathbb{F}_\beta$ at scale β resides within a background "space" $\mathbb{S}_\beta$ that is the statistical field $\mathbb{S}_\beta \Leftrightarrow \mathbb{F}_{\beta-1}$ at the adjacent lower scale $\beta-1$. For example, considering astrophysics $\beta = s$ at thermodynamic equilibrium, Wien wavelength of thermal oscillations of particles (stars) defines absolute thermodynamic temperature $T_s = \lambda_{ws}$ that leads to internal measures of spacetime $(\lambda_{ws}, \tau_{ws})$ associated with every "point" or "atom" of cosmological space $\mathbb{S}_g \Leftrightarrow \mathbb{F}_s$ at scale $\beta+1 = g$. Hence, for the study of dynamics of galaxies and their clusters in cosmology, *external* independent space and time coordinates (x_g, t_g) are defined in terms of the internal spacetime as [74]

$$(x_g, y_g, z_g) = (N_{xg}, N_{yg}, N_{zg})\lambda_{ws}, \quad t_g = N_{tg}\tau_{ws} \quad (34)$$

where $(N_{x\beta}, N_{y\beta}, N_{z\beta}, N_{t\beta})$ are independent numbers. The external time t_β in (34) is called Rovelli [69-71] *thermal time*.

According to (33)-(34), both internal and external space and time are quantized. The thermodynamic internal spacetime $(\lambda_{w\beta-1}, \tau_{w\beta-1})$ are *local sub-atomic dependent units* of space and time. On the contrary, the external space and time in (34) relate to *global* dynamics and are both *independent* and *irreversible*. Irreversibility of time or Eddington *arrow of time* is related to dissipations hence entropy according to second law of thermodynamics [74]. Irreversibility of space coordinates arises from its quantum nature through Heisenberg [75] *matrix mechanic* thus leading to noncommutative spectral geometry described by Connes [76]. The four dimensions $(x_\beta, y_\beta, z_\beta, t_\beta)$ constitute Poincaré [61] and Minkowski [77] pseudo-Riemannian space with three real space and one imaginary time dimensions.

Because time no longer explicitly appears in the field equations of general theory of relativity (GTR), some authors have claimed that time may be an illusion. However, as evident from their definitions in (34), external space and time retain their intuitive reality as measurable physically real entities. Also, and most importantly, external time and space in (34) are *totally different and independent coordinates that are not symmetric*. As noted by Verhulst [78], asymmetry of space and time was emphasized by Lorentz in his 1915 lecture at the Royal Academy of Sciences in Amsterdam:

"Why can we not speak of the ether instead of vacuum? Space and time are not symmetric; a material point can at different times be at different spots, but not in different places at the same time"

External space and time in (34) are only defined up to the critical atomic units $(N_{x\beta} = N_{t\beta} = 1)$. Below this atomic scale, one must move to statistical field at the next lower scale $\beta-1$ and employ (34) to obtain new space and time based on ruler and clock (spacetime) of scale $\beta-2$. At thermodynamic equilibrium between particle and radiation i.e., background "space" of Casimir [63] vacuum, following de Broglie's wavelength of matter waves, the definition of Boltzmann constant (27) naturally leads to the concept of *frequency of matter waves* [34]

$$\lambda_\beta = h / \bar{p}_\beta \quad \text{WAVELENGTH OF MATTER WAVE} \quad (35)$$

$$\nu_\beta = k / \bar{p}_\beta \quad \text{FREQUENCY OF MATTER WAVE} \quad (36)$$

where $\bar{p}_\beta = m_\beta v_{r\beta}$ is root-mean-square momentum of particle.

In usual quantum mechanics, unlike Heisenberg [79] uncertainty principle involving space and momentum, there is no uncertainty relation involving time and momentum as discussed by Schommers [80]

"Within usual quantum theory, space-coordinate and time are not symmetrical to each other, which is in contrast to the basic results of the special theory of relativity. In usual quantum

theory coordinates are operators and play the role of statistical quantities; time however does not behave statistically and remains a simple parameter when we go from classical to quantum mechanics. for this reason, there is no uncertainty relation for the time and energy within the quantum theory which would agree in its physical content with the coordinate and momenta."

However, because of the definition of Boltzmann constant (27), parallel to Heisenberg [79] *spatial uncertainty principle*, one can introduce *temporal uncertainty principle* [35,36]

$$\Delta\lambda_\beta \Delta\overline{p}_\beta \geq h \quad \text{SPATIAL UNCERTAINTY PRINCIPLE} \tag{37}$$

$$\Delta\nu_\beta \Delta\overline{p}_\beta \geq k \quad \text{TEMPORAL UNCERTAINTY PRINCIPLE} \tag{38}$$

Temporal uncertainty principle (38) places a limit on maximum resolution of time measurements.

Having defined space and time, the concept of atomic mass is examined next. Early literature concerning gravitational, electromagnetic, relativistic, aspects of concept of mass have been reviewed in an excellent book by Jammer [81]. The ambiguity associated with the meaning of mass was emphasized by Poincaré [82]

"There is no escape from the following definition, which is only a confession of failure: Masses are coefficients which it is found convenient to introduce into calculations".

Also, according to Einstein [83]

"Neither Newtonian nor the relativistic theory of gravitation has so far led to any advance in the theory of the constitution of matter".

Regarding another quotation from Einstein [84]

"Mass and energy are essentially alike; they are only different expressions for the same thing"

the fact that mass [kg] and energy [J] have *different dimensions* makes the understanding of such equivalence less intuitive.

By invariant definition (1) in the hierarchy of statistical fields shown in Fig. 1, one defines the "atomic" mass unit $\hat{m}_\beta$ at the scale β as the most-probable or Wien mass $m_{w\beta-1}$ of the lower scale $\beta-1$, that is the internal energy of the next lower scale $\beta-\tilde{2}$ For example, the classical atonic mass-unit *amu* at EAD scale in (29) is the most probable mass of ESD field

$$\hat{m}_a = amu_a = m_{ws} \tag{39}$$

that in turn is atomic internal energy of EKD field that is photon [33,34]

$$amu\ [\text{kg}] = \sqrt{hkc} = m_k c^2\ [\text{J}] = \hat{u}_k = 3kT_k \tag{40}$$

In (40), the origin of equivalence of *dimensions* of mass [kilogram] and energy [Joule] is fully revealed. Total internal energy or *electromagnetic mass* [34] of ideal photon gas becomes

$$M_{EM} = N(amu) = N\hat{u}_k = U_k = 3NkT_k \tag{41}$$

In his pioneering 1847 paper, Helmholtz [85] decomposed the total thermal energy of an ideal gas at thermodynamic equilibrium into two parts namely kinetic energy or *free heat* and potential energy or *latent heat* that were recently identified as *internal energy U* and *potential energy* pV [33,34]. Therefore, besides the internal energy (40), stability of particles requires an *external pressure* hence potential energy called Poincaré stress [34]. At thermodynamic equilibrium, due to equality of external and internal pressures, atomic potential energy of photon is

$$p_k \hat{v} = \hat{u}_k / 3 = kT_k \tag{42}$$

resulting in total potential energy, dark matter, or *gravitational mass* of photon [33,34]

$$M_{GR} = M_{DM} = p_k N\hat{v} = p_k V = NkT_k \tag{43}$$

Therefore, just like monatomic ideal gas, the total energy of photon gas at thermodynamic equilibrium is the sum of its electromagnetic and gravitational energy that is what Sommerfeld [86] called "total heat" or enthalpy

$$M_{\rm tot} = M_{\rm DE} + M_{\rm DM} = U_{\rm k} + {\rm p}V_{\rm k} = H_{\rm k} \qquad (44)$$

The ideal gas law (43) helps to reveal Joule-Mayer mechanical equivalent of heat and *mass-energy equivalence* discussed in an interesting but unfortunately little known 1904 study by Olinto de Pretto [87] who wrote:

"The vis viva $\mathrm{m}v^2$ *and the formula* $mv^2/8338$ *gives us, expressed in calories, such energy. Given that* $m = 1$ *and* v *equals three hundred thousand kilometers per second, that is 300 million meters, which would be the speed of light, also allowed for the ether, everyone will see that you get an amount of calories represented by 10794 followed by 9 zeros and that is over ten million million."*

containing de Pretto formula

$$\mathrm{mc}^2\,[\mathrm{kcal}] = \mathrm{mc}^2/8338 \;\; [\mathrm{Joule}] \qquad (45)$$

De Pretto number 8338 in (45) is numerically identical to the universal gas constant in (31) expressed in MKS units $\mathrm{R}^{\circ} = 8338$ [J/(kmol-m] [34]. This is because the factor of 2 in definition of temperature $T' = 2T$ [34] leads to modified value of Joule-Mayer mechanical equivalent of heat $J = 2J_{\rm c} = 2\times 4.169 = 8338$ [Joule/kcal] where the classical value $J_{\rm c} = 4.169 \approx 4.17$ kJ/kcal is the average of the two values $J_{\rm c} = (4.15, 4.19)$ reported by Pauli [68]. Hence, the ideal gas law in (43) gives

$$\mathrm{p_k}V = \textit{Work [J]} = (N^{\circ}\mathrm{k})(N/N^{\circ})T = \tilde{N}R^{\circ}T$$

$$= R^{\circ}\textit{[J/(kmol-m)]}(\tilde{N}T)\textit{[kmol-m]} \qquad (46)$$

By definition, thermal energy unit kcal corresponds to the amount of energy required to raise the temperature of one kg of water by unity hence $T = 1$ m

$$R^{\circ}\textit{[J/(kmol-m)]} = R^{\circ}\textit{[J/kcal]} = \\ = J \approx \textit{8338 [J/kcal]} \qquad (47)$$

and (47) assumes the classical form of mechanical equivalent of heat [34,68,86]

$$W = JQ \qquad (48)$$

Regarding mass energy equivalence, Okun [88] has argued that amongst the formulas

$$E = \mathrm{mc}^2 \quad \text{Poincaré} \qquad (49)$$

$$E_{\rm o} = \mathrm{mc}^2 \quad \text{Einstein} \qquad (50)$$

$$E_{\rm o} = \mathrm{mc}^2 \qquad (51)$$

$$E_{\rm o} = \mathrm{m_o c}^2 \qquad (52)$$

rather than the correct formula (49) introduced by Poincaré [59] in 1900, the formula (50) introduced later by Einstein [83] in 1905 is correct. Therefore, Okun [88] has in effect challenged the validity of the concept of relativistic mass of Lorentz [89]

$$\mathrm{m} = \mathrm{m_r} = \mathrm{m_o}/\sqrt{1-\mathrm{v}^2/\mathrm{c}^2} \qquad (53)$$

However, validity of (53) is evident from the fact that its multiplication with square of the speed of light results in the well-known relativistic energy transformation formula [59-62, 83-84]

$$E = E_{\rm o}/\sqrt{1-\mathrm{v}^2/\mathrm{c}^2} \qquad (54)$$

In formula (54), E refers to total or relativistic energy and hence involves total or relativistic mass of Lorentz (53), that upon expansion in powers of $\varepsilon = (\mathrm{v}/\mathrm{c})^2 << 1$ gives

$$E = \mathrm{m_r c}^2 \approx \mathrm{m_o c}^2 + \mathrm{m_o v}^2/2 + \\ +(3/8)\mathrm{m_o v}^4/\mathrm{c}^2 + \ldots \qquad (55)$$

However, expansion of the formula of Okun [88]

$$E = E_{\rm o} + E_{\rm kin} = \sqrt{\mathbf{p}^2\mathrm{c}^2 + \mathrm{m}^2\mathrm{c}^4} = \\ = \mathrm{mc}^2 + \frac{\mathbf{p}^2}{2\mathrm{m}} + \ldots \qquad (56)$$

to the next higher order of approximation gives

$$E = mc^2 + mv^2/2 - (1/8)mv^4/c^2 + ... \quad (57)$$

that is different from (55). The discrepancy is because (56) is equivalent to $E = E_o\sqrt{1+v^2/c^2}$ that does not agree with (54). Ironically, the quotation in Okun [88] paper contains the following statement by Einstein [84],

"The mass of a body is not a constant; it varies with changes in its energy"

The misunderstanding could in part be due to the fact that instead of Lorentz [89] relativistic mass transformation (53), Einstein [84] wrote

$$E = m/\sqrt{1-v^2/c^2} \quad (58)$$

Because both electromagnetic and gravitational mass in (41) and (43) are kinetic energy, the terminology "rest energy" used by Okun [88] could be modified to "stochastically stationary energy". This is because according to the present model, and in harmony with quantum mechanics, the state of rest or absolutely "static" state corresponds to $T_\beta = \lambda_{w\beta} = 0$ that is a singularity of the field inside black hole [90] as discussed in the last Section on cosmology.

In a related matter, it was stated by Einstein [84] and often repeated by others, that matter cannot be accelerated to the speed of light because by Poincaré-Einstein formula (54), this requires infinite amount of energy and hence is impossible. However, this is contrary to the fact that conversion of matter into light is very common occurrence in nature as anticipated by Newton [55] and in fact happens whenever we strike a match or light a candle!

In addition to (50) corresponding to *electromagnetic mass*, one must also account for potential energy or *gravitational mass* associated with *Poincaré stress* that is responsible for particle stability [34,37,90]. The infinite energy just mentioned is avoided because as matter is accelerated by heating, before reaching the speed of light $c = \sqrt{3}v_{wk+}$, it gets to the most probable or Wien speed [90]

$$\begin{aligned} v_\beta^2 \to v_{w\beta}^2 &= 2v_{w\beta+}^2 = \\ &= 2v_{r\beta+}^2/3 = 2c_\beta^2/3 \end{aligned} \quad (59)$$

For example, for photon gas $\beta = k$ in enclosures, substituting from (59) in (54) results in *finite* total energy given in 1905 by Hasenöhrl [91,92]

$$\begin{aligned} E &= E_o/\sqrt{1-v^2/c^2} \approx \\ &\approx m_o c^2(1+\frac{v^2}{2c^2}) = \frac{4}{3}m_o c^2 \end{aligned} \quad (60)$$

Total atomic energy of photon in (60) is the same as photon atomic enthalpy [36] obtained from (44)

$$\begin{aligned} \hat{h}_k = m_k u_k^2 = \hat{u}_k + p_k\hat{v} &= 4kT = \\ &= 4m_k v_{wk+}^2 = (4/3)m_k c^2 \end{aligned} \quad (61)$$

By (61), the superluminal longitudinal root-mean square speed of *atomic* photon becomes

$$u_k = 2v_{wk} = 2c/\sqrt{3} \quad (62)$$

The result (62) is in agreement with the ratio of *longitudinal* to *transverse* wave propagation speeds in elastic media given by Achenbach [93] as

$$\begin{aligned} C_L/C_T = \kappa &= \left[(\lambda+2\eta)/\eta\right]^{1/2} = \\ &= \left[\left(2(1-\nu)/(1-2\nu)\right)\right]^{1/2} = 2/\sqrt{3} \end{aligned} \quad (63)$$

with Lame constant $\lambda = -2\eta/3$, Poisson ratio $\nu = -1$ that marginally satisfies stability criteria $-1 < \nu < 1/2$ [47], $C_{L\beta}^2 = 2v_{w\beta}^2 = 4v_{w\beta+}^2$ by (3) since $< v_{w\beta}^2 >=< v_{w\beta}'^2 >$ due to Boltzmann principle of equipartition of energy, and $C_{T\beta}^2 = C_\beta^2 = 3v_{w\beta+}^2$. The Lame constant $\lambda = -2\eta/3$ satisfies Stokes assumption of vanishing *bulk modulus* or bulk viscosity (16). Also, as sum of two *stochastic velocities* according to (3), $C_L = 2C/\sqrt{3}$ is superluminal [94,95].

To conclude this Section, it is instructive to note that definition of (space, time, mass) of each statistical field within hierarchies of scales (Fig. 1)

$$(x_\beta, y_\beta, z_\beta, t_\beta, m_\beta) =$$
$$= [(N_{x\beta}, N_{y\beta}, N_{z\beta})\lambda_{w\beta-1}, N_{t\beta}\tau_{w\beta-1}, N_{m\beta}m_{w\beta-1}] \quad (64)$$

are relegated to the corresponding entities at the next lower scale by *independent* numbers $(N_{x\beta}, N_{y\beta}, N_{z\beta}, N_{t\beta}, N_{m\beta})$. Matter is considered to be infinitely divisible [90,96] because space, time, and mass must follow the philosophy of Anaxagoras [97]

"Neither is there a smallest part of what is small, but there is always a smaller, for it is impossible that what is should ever cease to be"

By infinite regression in (64), the reality of space, time, and matter hence epistemology of all existence (Fig. 1) reduces to *pure numbers*. This is in harmony with the perceptions of Pythagoras and Plato who ascribed all physical reality to *pure numbers* as eloquently described by Sommerfeld [98]

"Our spectral series, dominated as they are by integral quantum numbers, correspond, in a sense, to the ancient triad of the lyre, from which the Pythagoreans 2500 years ago inferred the harmony of the natural phenomena; and our quanta remind us of the role which the Pythagorean doctrine seems to have ascribed to the integers, not merely as attributes, but as the real essence of physical phenomena"

Physical Foundations of Quantum Mechanics and Theory of Relativity

Our discussion of physical foundation of quantum mechanics starts with an important quotation from Boltzmann, who anticipated quantum mechanics by about three decades, taken from his pioneering and unfortunately often neglected 1872 paper [99]

"We wish to replace the continuous variable x by a series of discrete values ε, 2ε, 3ε ... pε. Hence, we must assume that our molecules are not able to take up a continuous series of kinetic energy values, but rather only values that are multiples of a certain quantity ε. Otherwise, we shall treat exactly the same problem as before. We have many gas molecules in a space R. They are able to have only the following kinetic energies:

ε, 2ε, 3ε, 4ε, . . . pε

No molecule may have an intermediate or greater energy. When two molecules collide, they can change their kinetic energies in many different ways. However, after the collision the kinetic energy of each molecule must always be a multiple of ε. I certainly do not need to remark that for the moment we are not concerned with a real physical problem. It would be difficult to imagine an apparatus that could regulate the collisions of two bodies in such a way that their kinetic energies after a collision are always multiples of ε. That is not a question here."

Boltzmann significant contributions to the foundation of quantum mechanics include statistical definition of entropy and combinatoric methods that played central roles in Planck's [100] introduction of quantum of action and his theory of equilibrium thermal radiation.

In view of Fig. 1, following Boltzmann statistical mechanics [99], one considers a hierarchy of statistical quantum fields composed of (… gravitons, neutrinos, photons, electrons, atoms, …., planets, stars, galaxies, galactic clusters, …) particles. If one considers a universe initially composed of extremely large numbers of gravitons (tachyons) as ideal superluminal gas with estimated most probable graviton wave speed of $c_t = 7.7\times10^{13}\,c$ [37], the entire cosmos will be *causally connected*. As one lowers the temperature by expansion, the number of oscillators hence entropy S = 4NK [34] decreases until a critical temperature at which Cooper pairs of gravitons become stable. As temperature is further decreased, more and more clusters are formed until the state of equilibrium between ETD and END (equilibrium neutrino-dynamics) at $T_g = T_n$. Continued cooling of the field results in further condensations and formation of EKD, ESD, EAD, … statistical fields.

The hierarchy of statistical fields in the above thought experiment will have a corresponding hierarchy of Maxwell-Boltzmann distributions shown in Fig. 4

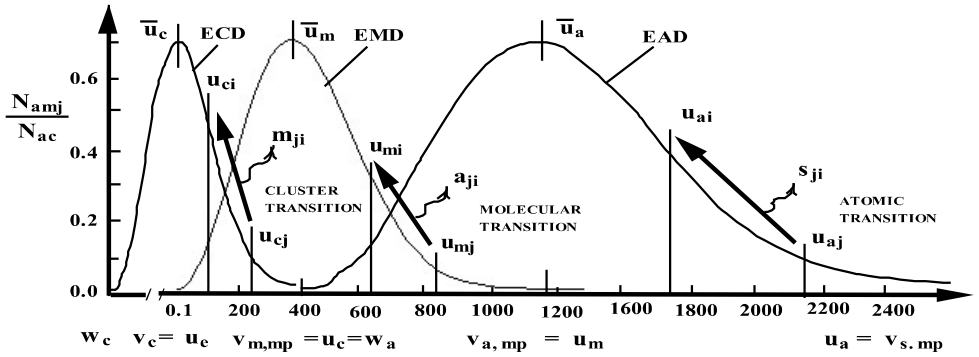


Figure 4 – Maxwell-Boltzmann speed distribution viewed as stationary spectra of cluster sizes for ECD, EMD, and EAD scales at T = 300 m [34].

In Fig. 4, the atomic mass and mass of most probable element of adjacent field are related as $\hat{m}_\beta = m_{w\beta-1}$ in accordance with (40). The smallest element or "atom" of each statistical field is a *composite boson* formed by "Cooper pair" of *most probable element* of the lower scale [36]. The atom is smallest element and is therefore the most stable element of the field. Internal energy of all elements will be the same due to Boltzmann principle of equipartition of energy [36]. Therefore, at *thermodynamic equilibrium*, each statistical field shown in Fig. 4 is composed of a spectrum of cluster sizes, also called energy levels, containing a corresponding spectrum of atoms that are under constant random transitions between different energy levels. Transitions of "atoms" between different clusters, energy levels, result in emission (absorption) of "sub-particle" in accordance with Bohr frequency relation $\Delta\varepsilon_{ji\beta} = h(\nu_{j\beta} - \nu_{i\beta})$ [101]. Atomic transitions between different clusters is in harmony with recent *cellular automaton* model of quantum mechanics [102].

Schematic diagrams of atomic transitions between two different clusters, de Broglie [2] wave packets, are shown in Fig. 5.

For example, at cosmic scales $\beta = g$, transition of a galaxy from a small rapidly oscillating cluster (high-energy-level j) to a large slowly oscillating cluster (low-energy-level i) results in emission of a star s_{ij} as a sub particle of cosmic field [37]. Hence, according to the new physical foundation of quantum mechanics [37], Bohr *stationary states* [101] correspond to *stochastically stationary* sizes of atomic clusters, *energy levels*, shown in Fig. 5. Clusters or de Broglie [3] wave packets are stabilized by an external pressure called Poincaré stress [36]. In Fig. 5, it is noted that transition energies $\Delta\varepsilon_{ji\beta} = h(\nu_{j\beta} - \nu_{i\beta})$ are double-indexed in accordance with Heisenberg [75] *matrix mechanics* that naturally lead to *noncommutative spectral geometry* of Connes [76].

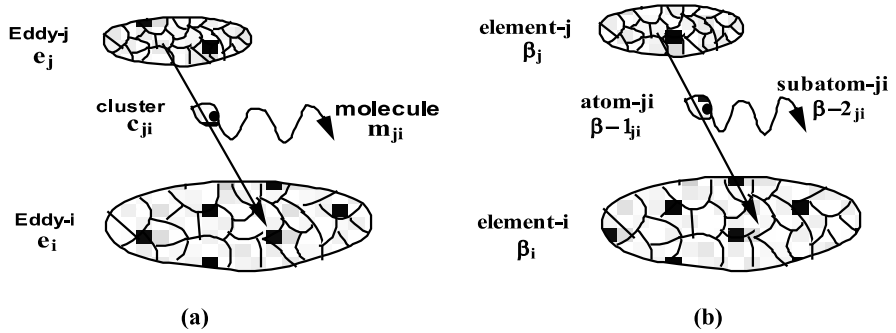


Figure 5 – (a) Transition of cluster c_{ij} from eddy-j to eddy-i leading to emission of molecule m_{ij}. **(b)** Generalized transition [36].

Under the assumption of potential incompressible flow, invariant Cauchy equation of motion (6) results in invariant Bernoulli equation that through Hamilton-Jacobi equation of classical mechanics leads to invariant Schrödinger equations [37,103]

$$\frac{\hbar^2}{2m_\beta}\nabla_\xi^2\psi_\beta + (\tilde{H}_\beta - \tilde{V}_\beta)\psi_\beta = 0 \qquad (65)$$

$$i\hbar\frac{\partial\Psi_\beta}{\partial t_\beta} + \frac{\hbar^2}{2m_\beta}\nabla_\xi^2\Psi_\beta - \tilde{V}_\beta\Psi_\beta = 0 \qquad (66)$$

with wave function defined as [37]

$$\Psi_\beta(\xi,t) = \rho^{1/2}\Phi'_\beta(\xi,t) = \rho^{1/2}\, e^{ik_r\xi}\, e^{-i\tilde{H}t_\beta/\hbar} \qquad (67)$$

such that $\Psi_\beta\Psi_\beta^* = \rho_\beta$ accounts for *particle localization* in accordance with classical results [104]. Therefore, for incompressible potential flows, invariant Schrödinger (65-67) governs the dynamics of Heisenberg-Kramers virtual oscillators [105] in all statistical fields shown in Fig. 1. Also, in harmony with perceptions of de Broglie [2,3], hierarchies of statistical fields in Fig.1 are embedded *wave-packets* that guide motion of "particles" as schematically shown in Fig. 6 at EUD, EGD, …, EMD, and EAD scales.

The direction of peculiar velocity of particle is that of the gradient of its velocity potential $V'_\beta = -\nabla\Phi'_\beta$. However, since the direction of atomic, mean, and peculiar translational velocities in (3) are the same, quantum mechanics wave function (43) *guides* the motion of particle in accordance with de Broglie-Bohm interpretation of quantum mechanics [3,37,106].

As one moves to smaller scales, most probable element is composed of an entire statistical field (see Fig. l), one can define a new velocity potential $\Phi'_{\beta-1}$, thereby a new wave function $\Psi_{\beta-1} = \rho^{1/2}\Phi'_{\beta-1}$. The cascade of particles as singularities embedded in *guidance waves* shown in Fig. 6 is in exact agreement with the perceptions of *de Broglie* concerning interactions between the particle and the *"hidden thermostat"* [3]

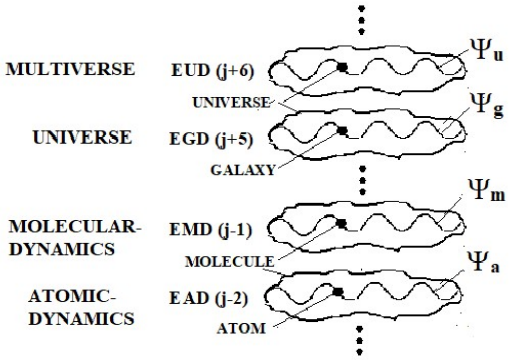


Figure 6 – Hierarchy of quantum mechanics wave functions m_{ij} or de Broglie guidance waves from multiverse (EUD) to atomic (EAD) scales [90].

"Here is another important point. I have shown in my previous publications that, in order to justify the well-established fact that the expression $|\Psi(x,y,z,t)|^2\, d\tau$ *gives, at least with Schrodinger' equation, the probability for the presence of the particle in the element of volume d at the instant t, it is necessary that the particle jump continually from one guidance trajectory to another, as a result of continual perturbation coming from subquantal milieu. The guidance trajectories would really be followed only if the particle were not undergoing continual perturbations due to its random heat exchanges with the hidden thermostat. In other words, a Brownian motion is superposed on the guidance movement. A simple comparison will make this clearer. A granule placed on the surface of a liquid is caught by the general movement of the latter If the granule is heavy enough not to feel the action of individual shocks received from the invisible molecules of the fluid, it will follow one of the hydrodynamic streamlines. If the granule is a particle, the assembly of the molecules of the fluid is comparable with the hidden thermostat of our theory, and the streamline described by the particle is its guiding trajectory. But if the granule is sufficiently light, its movement will be continually perturbed by the individual random impacts of the molecules of the fluid. Thus, the granule will have, besides its regular movement along one of the streamlines of the global flow of the fluid, a Brownian movement which will make it pass from one streamline to another. One can represent Brownian movement approximately by diffusion equation of the form* $\partial\rho/\partial t = D\Delta\rho$, *and it is interesting to seek, as various authors have done recently, the value of the coefficient D in the case*

of the Schrodinger eq nation corresponding to the Brownian movement.

I have recently studied [14] the same question starting from the idea that, even during the period of random perturbations, the internal phase of the particle remains equal to that of the wave. I have found the value $D = \hbar/3m$, *which differs only by a numerical coefficient from the one found by other authors.*

This concludes the account of my present ideas on the reinterpretation of wave mechanics with the help of images which guided me in my early work. My collaborators and I are working actively to develop these ideas in various directions. Today, I am convinced that the conceptions developed in the present article, when suitably developed and corrected at certain points, may in the future provide a real physical interpretation of present quantum mechanics."

When particles act as composite bosons [107] by forming Cooper-pairs, following classical methods [108, 109], quantum mechanics wave function may be expressed as products of translational, rotational, vibrational, and potential wave functions [110]

$$\Psi_\beta = \Psi_{t\beta}\Psi_{r\beta}\Psi_{v\beta}\Psi_{p\beta} = \Psi_{t\beta}\Psi_{r\beta}\Psi_{v\beta}\Psi_{\beta-1} = \ldots \quad (68)$$

At cosmological scale Ψ_g, the wave-particle duality of galaxies is evidenced by their observed quantized red-shifts [111]. Therefore, scale-invariance of the model (Fig. 1) and (68) leads to hierarchy of embedded statistical fields with translational TKE, rotational RKE, vibrational VKE kinetic energy and potential energy PE resulting in energy cascade shown in Fig. 7

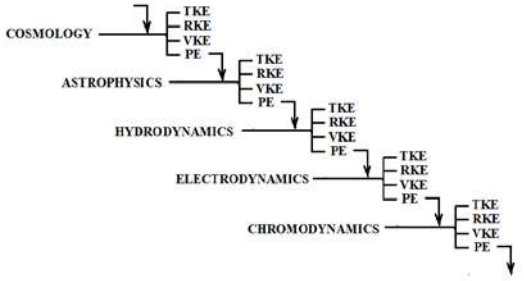


Figure 7 – Cascades of kinetic energy TKE, RKE, VKE (dark energy), and potential energy PE (dark matter) from cosmic to photonic scales [90].

Besides its complex nature that allows for normalization thus satisfying Born [112] probabilistic interpretation, the quantum mechanics wave function (67) resolves the problem of *empty waves* or what Einstein referred to as *Gespensterfelder* (Ghost fields) discussed by Selleri [113]. This problem arises from the fact that in usual quantum mechanics, the wave function is considered to carry neither energy nor momentum hence the name "empty wave" by Selleri [113].

"Whatever the approach, the question is always: How can one hope to reveal the presence of a wave which does not carry observable amounts of energy and momentum?"

According to the definition in (67), the complex wave function acts as "hidden variable" since it is related to an objectively real wave associated with particle *peculiar velocity.* Peculiar velocity of particles, although appreciated in cosmology, have been neglected in classical fluid mechanics. By (67), gradient of wave function through particle peculiar velocity hence Poincaré stress [35. 36] "guides" the motion of particle in harmony with de Broglie-Bohm "*pilot wave"* interpretation [3, 106] of quantum mechanics.

Although the validity of quantum mechanics is fully established, there are many interpretation problems confronting physical foundation of quantum mechanics seven of which were discussed in a recent investigation [37]

1. The nature of wave function and its probabilistic interpretation.
2. Wave-particle-duality.
3. Particle-particle entanglement.
4. Double-slit experiment.
5. EPR and problem of action-at-a-distance.
6. Quantum-jumps and trajectory problems.
7. Schrödinger cat.

In the following, the new paradigm of physical foundation of quantum mechanics [37] is applied to present very brief response to the interpretation problems identified in the list (1-7) above.

1. Schrödinger cat paradox is much more complex since it involves 1. By definition of wave function (67), both real or objective part namely density hence particle localization as well as its statistical complex or subjective part hence Born [112] probabilistic interpretation become self-evident.

2. In (1), "atom" or particle of scale β is by definition the most probable cluster or de Broglie "wave packet" $\mathbf{u}_\beta = \mathbf{v}_{w\beta-1}$ of the lower scale $\beta-1$ (see Fig. 6), thus resolving wave-particle duality problem [114].

3. By definition (1), particles or wave-packets are local "condensation" of the participating background field i.e., physical space in harmony with modern QFT. Therefore, separation of two entangled "particles" could be viewed as separation of two droplets, Wheeler-Bohr model of particle, schematically shown in Fig. 8.

Clearly, initially entangled droplets will retain correlation due to the external participating field surrounding them regardless of their separation distance. Thus, separated particles communicate through *"ontological"* [102] sub-atomic background field accounting for what appears as action-at-a- distance or Bell non-locality.

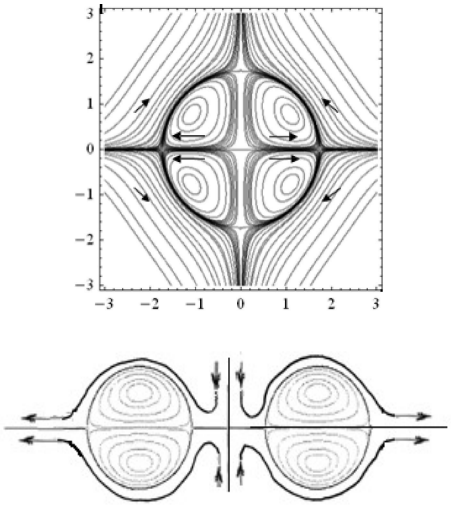


Figure 8 – Hydrodynamic model of fission of two hemispherical vortices into two Hill spherical vortices [37].

4. Since physical space or Casimir vacuum is identified as a compressible fluid, particle motion results in sub-photonic waves in the fabric of space or ether leading to interference patterns schematically shown in Fig. 9.

5. The EPR paradox is resolved due to particle entanglement in item-3 above as well as super-luminal tachyonic signals by *gravitational radiation* with estimated speed of $c_t = 7.7\times10^{13}c$ [37] making the entire universe causally connected. Possible superluminal velocities in ether to resolve Bell non-locality problem has been emphasized by Eberhard [94].

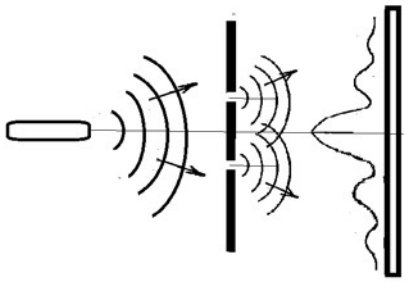


Figure 9 – Double-slit experiment and interference patterns due to formation of de Broglie guidance waves in Casimir vacuum.

6. Each statistical field is composed of a spectrum of clusters containing *integral numbers* of *identical* atoms that are nonetheless distinguishable because of their different energy. Hence, any atom of cluster j can make transition *through any arbitrary trajectory* to cluster i and emit a sub particle with energy ε_{ji} with exactly the same final results (see Fig. 5). Because probability of any intermediate energy level between level-j and level-i is zero, transitions can only occur through *quantum jumps* (see Fig. 4). At higher energies, higher frequencies, the spacing between energy levels decreases and the spectrum approaches a continuum corresponding to "bremsstrahlung" radiation for photon gas.

7. Schrödinger cat paradox is much more complex since it involves phenomena of life and death that are not understood. In a recent study [89], it was noted that because of the well know decoupling of radiation and matter fields in cosmology, one cannot rule out possible decoupling at sub-photonic scales within hierarchies of wave functions in (68)

$$\cdots\Psi_{\beta=d+2}\Psi_{\beta=d+1} \Leftarrow \quad \Rightarrow \Psi_{\beta=d}\Psi_{d-1}\Psi_{d-2}\cdots$$

at the moment of death $t = t_f$ or what Hegel called *the moment when the spirit transcends temporality* [115]. It is most fascinating that similar perception about possible relationship between material and non-material world was discussed by Lorentz in his letter of 1915 to the theologian H. Y. Groenewegen qouted in a recent wonderful book about Lorentz' life by Kox and Schatz [116]

"The single observation that the spirit of different people can understand each other, that the emotions and opinions of one are not indifferent to another, must lead us to the assumption of a connection between them, which many of us would like best to imagine in such a form that all these individual spirits are part of one great whole, a world spirit or deity. [...] But if the spirit is part of a great whole, just like the body is part of the entire material world, then naturally one arrives at the generalization that, as a rule, every event in the psychic realm responds to a change in the material realm. [...] The concept one arrives at in this way is that the spiritual and the material are inextricably connected, that they are the two sides of the same thing, that the material world is a representation of the world spirit"

Next, the impact of the model on theory of relativity will be briefly discussed. Since *physical space* or ether is identified as compressible ideal gas in harmony with perceptions of Huygens [54] and Planck [58], its density when isentropically brought to rest will be given by [117]

$$\rho = \frac{M}{V} = \rho_o \left[1 + \frac{\gamma - 1}{2} \frac{v^2}{c^2} \right]^{\frac{1}{\gamma - 1}} = \\ = \frac{M}{V_o} \left[1 + \frac{\gamma - 1}{2} Mi^2 \right]^{\frac{1}{\gamma - 1}} \tag{69}$$

where $\gamma = c_p / c_v$ is Poisson adiabatic index. For Casimir [63] vacuum or photon gas $\gamma = 4/3$ and assuming that *transverse coordinates* do not change [118] for one dimensional compressible flow, (69) leads to Lorentz-FitzGerlad contraction [33, 36, 117]

$$V = V_o \sqrt{1 - v^2 / c^2}, \quad \ell = \ell_o \sqrt{1 - v^2 / c^2}, \\ \rho = \frac{\rho_o}{1 - v^2 / c^2} \tag{70}$$

According to (70), compressibility of physical space accounts for Lorentz-FitzGerald contractions and provides a *causal* explanation of such relativistic effects as noted by Pauli [118]. When Michelson number *Mi* = v/c approaches unity, relativistic effects due to compressibility of ether i.e., physical space, become appreciable. Therefore, two different and experimentally distinguishable paradigms of special theory relativity (STR) in harmony with ideas of Darrigol [119] and Galison [120] were recently presented [117]

(A) Poincaré-Lorentz
Dynamic Theory of Relativity
Space and time (x, t) are altered due to causal effects of motion on the ether.

(B) Einstein
Kinematic Theory of Relativity
Space and time (x, t) are altered due to the two postulates of relativity:

1- Velocity of light is a universal constant independent of the motion of its source.
2- The laws of physics do not change form for all inertial frames of reference.

Although above postulates 1 and 2 are extremely accurate, strictly speaking they are not valid. This is because the speed of light is not constant but a function of temperature of Casimir [63] vacuum and hence decreases extremely slowly at cosmic time scales (eons) with expansion of universe. The second postulate is not valid because all inertial frames are *distinguishable* through measurements with respect to *stochastically stationary* isotropic cosmic microwave background radiation temperature of Penzias-Wilson [121, 122, 123] as emphasized by Prokhovnik [124].

Poincaré [59, 60] and Lorentz [89] introduced their *dynamic theory of relativity* in order to explain the relativistic Lorentz-FitzGerald contractions *causally* as emphasized by Pauli [118] and described in more details in an excellent study by Hirosige [125]

"In contrast to Einstein, Lorentz, Poincaré, and most other contemporary physicists saw the Michelson-Morley experiment as one of the most urgent problems requiring their theoretical efforts."

And similarly [125]

"Poincaré anticipated the principle of relativity as an empirical law, looking forward to a theory which could explain or prove the principle."

Also, Helmholtz is known to have developed very complicated equations to describe motion of atoms of ether according to Lorentz [126]. It is therefore natural to search for a *causal* relativistic theory of gravitation based on solution of conservation equations (4)-(7) for Huygens' [54] compressible ether as discussed in Section 6 on cosmology. Accordingly, one expects the problem to primarily involve *scalar* and *vector* fields and shock waves in unsteady gas dynamics. For example, the analogy between fluid mechanics of air and ether suggests that supersonic $Ma > 1$ (superluminal $Mi > 1$) flow of air (ether) leads to the formation of *Mach* (*Poincaré-Minkowski*) cone that separates the zone of sound (light) from the zone of silence (darkness) [37, 117] as schematically shown in Fig. 10.

Einstein [127] on the other hand, introduced his *kinematic theory of relativity* based on two *postulates* in order to account for asymmetries in Maxwell [57] theory of electromagnetism when applied to moving bodies. The fundamental limitation of such scientific theory based on *postulates* is that it cannot reveal the true underlying causes of the observed relativistic effects such as space and time interactions. In other words, the kinematic theory based on postulates (1)-(2) in paradigm B above cannot address the exact reasons and the dynamic mechanisms that are responsible for relativistic interdependence of space and time.

It is also important to emphasize that dynamic versus kinematic theories of relativity in paradigms (A) versus (B) above are *experimentally distinguishable*. This is because higher /lower local densities will be measured in front/rear by an observer moving with respect to *stochastically-stationary* compressible ether as schematically shown in Fig. 11.

Since Lorentz-FitzGerald contractions are *physically real*, they only agree with predictions of kinematic theory of relativity locally in one-dimensional experiments. For the same reasons, Einstein principle of equivalence is only locally true. It is expected that future multi-dimensional experiments will detect *transverse* topological change of space curvature due to *three-dimensional* nature of compressibility effects (Fig. 11) in accordance with predictions of paradigm (A) above. Furthermore, in dynamic theory of relativity, spacetime is governed by thermodynamic temperature as discussed in Section 3. For example, in the classical problem of *twin paradox*, different times experienced by the twins are real and attributed to the different *biological reaction rates* in their bodies induced by the compressibility of physical space [128] governed by dynamic theory of relativity of Poincaré [59,61] and Lorentz [89].

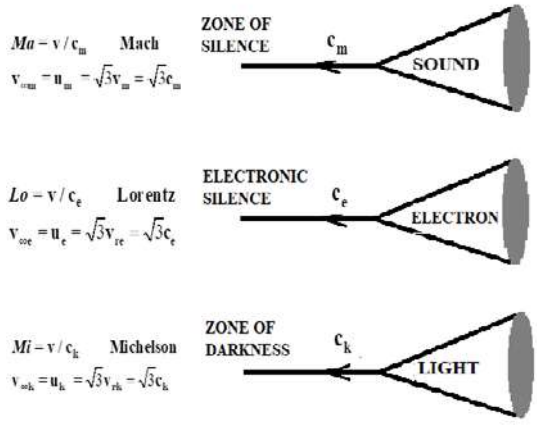


Figure 10 – "Supersonic" flows at molecular-, electro-, and chromo-dynamics scales leading to the formation of Mach, Lorentz, and PoincaréMinkowski cones [37].

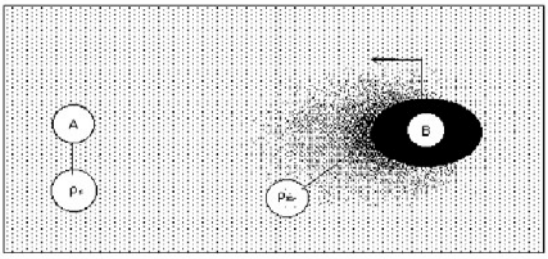


Figure 11 – Density of a medium as measured by an observer that is (A) stationary (B) moving with respect to the medium.

It is instructive to compare the singular behavior of relativistic transformations of mass (53) by Lorentz and energy (54) by Poincare-Einstein in Section 3 with the singularity in shock density formula (70) of gas-dynamics. It is well known that when Mach number $Ma = v / c_m$ exceeds unity, a shock wave occurs leading to transition from supersonic flow $v > c_m$ at equilibrium cluster-dynamics ECD to subsonic flow $v < c_m$ at equilibrium molecular dynamics EMD across the shock wave [37]. Naturally, one anticipates similar shock waves to occur at much higher velocities, when Lorentz

number $Lo = \mathrm{v}/\mathrm{c}_e$ (Michelson number $Mi = \mathrm{v}/\mathrm{c}$) exceeds unity, leading to transition $\mathrm{v} > \mathrm{c}_e \rightarrow \mathrm{v} < \mathrm{c}_e$ ($\mathrm{v} > \mathrm{c}_k \rightarrow \mathrm{v} < \mathrm{c}_k$) associated with ESD and EKD scales as shown in Fig. 10 [37]. Therefore, superluminal flow of tachyons [129] (gravitons) becomes understandable since critical root-mean-square wave velocities $(\mathrm{c}_m, \mathrm{c}_e, \mathrm{c}_k)$ in (Ma, Lo, Mi) are average speeds of much smaller "subatomic" particles at the next lower scale. By (62) in Section 3, the maximum "atomic" photon speed shown in Fig. 10 becomes $\mathrm{u}_{\infty k} = \sqrt{3}\mathrm{u}_k = 2\mathrm{c}$.

In regards to Lorentz perceptions concerning reality of ether and Lorentz-FitzGerald contractions it is important to quote directly from the treatise of Lorentz [126]

"I cannot speak here of the many highly interesting applications which Einstein has made of this principle, His results concerning electromagnetic and optical phenomena (*leading to the same contradiction with Kaufmann's results that was pointed out in* §179[1]) *agree in the main with those which we have obtained in the preceding pages, the chief difference being that Einstein simply postulates what we have deduced, with some difficulty and not altogether satisfactorily, from the fundamental equations of the electromagnetic field. By doing so, he may certainly take credit for making us see in the negative results of experiments like those of Michelson, Rayleigh and Brace, not a fortuitous compensation of opposing effects but the manifestation of a general and fundamental principle.*

Yet, I think, something may also be claimed in favor of the form which I have presented the theory. I cannot but regard the ether, which can be the seat of electromagnetic field with its energy and its vibrations, as endowed with a certain degree of substantiality, however different it may be from all ordinary matter. In this line of thought, it seems natural mot to assume at starting that it can never make any difference whether a body moves through the ether or not, and to measure distances and lengths of time by means of rods and clocks having fixed position relative to the ether.

It is important to examine the reasons for the dismissal of the prior ether theories [65]. As stated in [36], classically, ether was considered as a medium that occupied space in harmony with perceptions of Aristotle [53], Huygens [54], Newton [55], Euler [56], Maxwell [57], Lorentz [58], Poincaré [59-62], de Broglie [3], Casimir [63], and Dirac [64] amongst many others. However, in his kinematic theory of relativity Einstein [127] found ether to be "*superfluous*" since it is undetectable and could be replaced by abstract concept of *spacetime* instead of classical Newtonian absolute space and time. Therefore, many physicists were convinced that Einstein kinematic theory of relativity proved non-existence of ether and the null result of Michelson-Morely [130] experiments. Other physicists were convinced because of mathematical beauty of Poincaré-Minkowski four dimensional spacetime. As a result, Lorentz-FitzGerald contraction as a causal explanation of null result of Michelson-Morely [130] experiment was dismissed as ad hoc assumption. The author agrees with Dirac [131] that mathematical beauty and symmetry in the expression of the laws of Nature are extremely important. However, perhaps it is also important to emphasize that even when mathematically beautiful theories have been established, in the absence of a clear *physical and intuitive* explanations to compliment such theories, the search for alternative theories as well as different interpretation of the existing theory should continue with open mind. Further details about significant role of ether in research programs of Lorentz and Poincaré are discussed in the excellent study of Hirosige [125].

It appears that distinction between dynamic (A) versus kinematic (B) paradigms identified above was known to Poincaré based on fundamental principles as suggested by the lecture he delivered in London in 1912 shortly before he died [132]

"Today some physicists want to adopt a new convention. It is not that they are constrained to do so; they consider this new convention more convenient; that is all. And those who are not of this opinion can legitimately retain the old one in order not to disturb their old habits. I believe, just between us, that this is what they shall do for a long time to come,"

The perceptions of Poincaré concerning relativity theory are known to be also shared by Lorentz who stated in a 1915 lecture at the Royal Academy of Sciences in Amsterdam [78]

I could point out to you [if I had more time] how Poincaré in his study of dynamics of electron, about the same time as Einstein, formulated many ideas that are characteristic for his theory, and also formulated what he calls "le postulat de relativité"

It is clear that because of introduction of privileged frame of ether, Galilean transformation will be always violated because of the fact that all inertial frames are in reality not precisely equivalent. Therefore, the problem of relativity reduces to the problem of simple allowance for the compressibility of the manifold within which dynamic processes occur**.** From the perspective of the present field theory, the notion of STR then states that if one desired to arrive at a constant and *finite propagation velocity* within a *compressible medium*, but erroneously assumes that the medium within which the dynamics occur is *incompressible***,** then the application of STR will modify the local space and time such as to compensate for this erroneous assumption. The apparent artificiality of such a procedure is because physical space for relativistic events is obviously a compressible medium since the propagation velocity of light is finite.

In closing this Section, it is instructive to note that derivation of invariant Schrödinger equation from invariant Bernoulli equation was based on two important assumptions, namely (a) incompressible flow (b) potential flow [37]. Relaxation of assumption (a) hence inclusion of compressibility effects means including relativistic effects (70) as discussed above. Relaxation of assumption (b) implies presence of viscous effects i.e., vorticity hence iso-spin (8) thus requiring consideration of modified Navier-Stokes (19) rather than Cauchy equation of motion (6). Interestingly, it was precisely the viscous equation of motion (19) that was employed in a recent investigation [35] to derive Dirac [133] relativistic wave equation of quantum field theory (QFT)

$$i\hbar\left(\frac{1}{c}\frac{\partial \Psi_{D\beta}}{\partial t_{\beta}}+\alpha_{j\beta}\frac{\partial \Psi_{D\beta}}{\partial x_{j\beta}}\right)+(\alpha_{m\beta}m_{\beta}c)\Psi_{D\beta}=0 \quad (71)$$

Dirac [133] arrived at his relativistic wave equation (71) through shear genius being guided by his superb mathematical intuition. Therefore, the physical basis of his relativistic wave equation (71) and hence QFT remains abstract and mysterious. The simple derivation of Dirac equation in [35] may help the understanding of physical foundation of this important equation. It is also important to note that as long as $\nabla\times\mathbf{v}_{\beta}=\nabla\times\mathbf{u}_{\beta}$ is true such that $\nabla\times\mathbf{v}'_{\beta}=0$ by (3), Dirac wave function $\Psi_{D\beta}$ remains well defined insuring validity of (71) for particles with spin such as election as is to be expected. Finally, Dirac [133] anticipated that the 4×4 tensors (α_j,α_m) in (71) may be related to *internal coordinates*. Hence, in harmony with Dirac's mathematical intuition, it is reasonable to suspect that tensors (α_j,α_m) are somehow connected to the four-coordinates $(z_{\beta},\theta_{\beta},r_{\beta},z'_{\beta})$ and the corresponding four-velocities $(\mathbf{v}_{z\beta},\mathbf{v}_{\theta\beta},\mathbf{v}_{r\beta},V'_{z\beta})$ discussed after Fig. 15 of the following Section.

Physical Foundation of Electromagnetism and Invariant Maxwell Equations

It is well known that, following Faraday's intuitive concept of line of force, Maxwell developed equations of electrodynamics guided by analogy with hydrodynamics of a "fictitious fluid" called ether. In particular, the important connection between magnetic field and fluid rotation was emphasized by Maxwell as described in his 1891 treatises on electrodynamics [57]:

"Whatever light is, at each point of space there is something going on, whether displacement, or rotation, or something not yet imagined, but which is certainly of the nature of a vector or directed quantity, the direction of which is normal to the direction of the ray. This is completely proved by the phenomena of interference."

"The only resemblance which we can trace between a medium through which circularly polarized light is propagated, and a medium through which lines of magnetic force pass, is that in both there is a motion of rotation about an axis. But here the resemblance stops, for the rotation in the optical phenomena is that of the vector which represents the disturbance. This vector is always perpendicular to the direction of the ray, and rotates about it a known number of times in a second. In a magnetic phenomenon, that which rotates has no properties to which its sides can be distinguished, so that we cannot determine how many times it rotates in a second."

According to Maxwell [57], Faraday conceived such lines of force as individually rotating lines in accordance with the Rankine hypothesis of molecular vortices. We quote another statement by Maxwell [57]

"If we adopt Ampere's theory, we consider a magnet not as a continuous substance, the magnetization of which varies from point to point according to some easily conceived law, but as a multitude of molecules, within each of which circulates a system of electric currents, giving rise to a distribution of magnetic force of extreme complexity, the direction of the force in the interior of the molecule being generally the reverse of that of the average force in its neighborhood, and the magnetic potential, where it exists at all, being a function of as many degrees of multiplicity as there are molecules in the magnet."

Following Dirac [131], a hydrodynamic model of Faraday line of force was recently introduced [90] on the basis of flow field surrounding spinning spherical particles in an otherwise stationary background fluid as schematically shown in Fig. 12

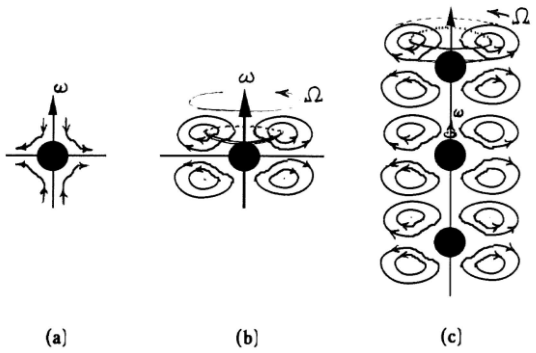


Figure 12 – Schematic model (a) flow near a spinning particle (b) locally conserved flow streamlines (c) formation of Faraday line of force from a row of co-spinning particles and the associated vortex field within the subquantum background fluid [90].

In general, interactive forces between spinning particles within participating background fluid is very complex and depends on the orientation of iso-spins of particles as schematically shown in Fig. 13

As discussed in [90], due to such hydrodynamic forces, spinning particles such as electrons form a chain composed of alternative particle/anti-particle or electron/positron called *hydrodynamic model of Faraday lines of force* schematically shown in Fig. 14

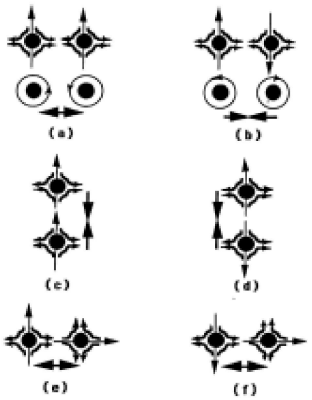


Figure 13 – Some possible interactions between spinning spherical particles within a participating background fluid. (a) parallel up-up (b) parallel up-down (c) co-axial up-up (d) co-axial up-down (e) anti-parallel up-right (f) anti-parallel down-right.

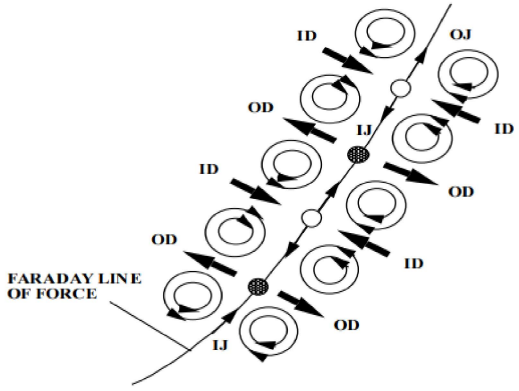


Figure 14 – Faraday line of force as electron (black) and positron (white) string with inflow jet (IJ) of one matching the outflow jet (OJ) of its neighbor. Also shown are alternating outflow (OD) and inflow discs (ID) [90].

The breakage of a line of force corresponds to creation of an electron and positron pair as described by Dirac [131]. It is reasonable to anticipate that what is known as "charge" is related the magnetic field and hence to iso-spin of particles with positive and negative charges associated with the sense of rotation of matter and anti-matter particles. At much smaller scales, such a hydrodynamic model of Faraday lines of force is in harmony with chains of quark-antiquark in stochastic chromodynamics "*like beads on a necklace*" to borrow the description by 't Hooft [134].

Therefore, in harmony with hydrodynamic model of Faraday line of force [89] and following Maxwell [57] and Lorentz [126], generalized model of electrodynamics is expressed by invariant Maxwell equations

$$\nabla.\mathbf{D}_\beta = \rho_\beta \tag{72}$$

$$\nabla \times \mathbf{E}_\beta = -\frac{\partial \mathbf{B}_\beta}{\partial t_\beta} \tag{73}$$

$$\nabla.\mathbf{B}_\beta = 0 \tag{74}$$

$$\nabla \times \mathbf{H}_\beta = \mathbf{J}_\beta + \frac{\partial \mathbf{D}_\beta}{\partial t_\beta} \tag{75}$$

Also, following Maxwell, at any scale one introduces a displacement vector $\mathbf{D}_\beta$ defined by (72) such that the continuity equation (4) in the absence of reactions leads to

$$\nabla.(\rho_{i\beta}\mathbf{v}_\beta) + \frac{\partial \rho_{i\beta}}{\partial t_\beta} = \nabla.(\mathbf{J}_\beta + \frac{\partial \mathbf{D}_\beta}{\partial t_\beta}) = 0 \tag{76}$$

Therefore, the quantity within parathesis must be curl of a vector called $\mathbf{H}_\beta$ leading to (75).

According to classical theory of electromagnetism [57, 126, 135-137], electromagnetic waves are *transverse waves*. However, following Huygens' [54] analogy between propagation of sound in air and light in ether, it is suggested that light waves also possess a longitudinal polarization associated with periodic *compression* and *rarefaction* of physical space. The neglect of longitudinal component could be due to exceedingly small thickness of chromodynamic shock. For example, assuming shock thickness to be of the order of size of nucleus $\delta \approx 10^{-15} m$, time scales of fluctuations in speed of light will be of order $\delta / c \approx 10^{-23} s$ and hence difficult to detect. Longitudinal wave could account for finite gravitational mass of photon (28) in harmony with Higgs mechanism [138]. With vector potential $\mathbf{A}_\beta$ herein identified as velocity field, magnetic field strength $\mathbf{B}_\beta$ is related to vorticity $\boldsymbol{\omega}_\beta$ by

$$\mathbf{B}_\beta = -\nabla \times \mathbf{A}_{z\beta} = -\nabla \times \mathbf{v}_{z\beta} \tag{77}$$

that results in Maxwell equation (74). The reason for the choice of negative sign in (77) is to have positive sign for the time derivative of electric field strength defined as longitudinal acceleration

$$\mathbf{E}_{z\beta} = \frac{\partial \mathbf{A}_{z\beta}}{\partial t_\beta} = \frac{\partial \mathbf{v}_{z\beta}}{\partial t_\beta} = \mathbf{a}_{z\beta} \tag{78}$$

Hence, by (76) and (78), the curl of $\mathbf{E}_{z\beta}$ gives

$$\nabla \times \mathbf{E}_{z\beta} = \frac{\partial}{\partial t_\beta}(\nabla_\beta \times \mathbf{A}_{z\beta}) = -\frac{\partial \mathbf{B}_\beta}{\partial t_\beta} \tag{79}$$

that is Maxwell second equation (73).

The sign convention in (77) leading to (79) ensures that a charge in electric field $\mathbf{E}_{z\beta}$ experiences a force $\mathbf{F}_{z\beta}$ in the same direction as the momentum since

$$\mathbf{F}_{z\beta} = \frac{\partial \mathbf{J}_{z\beta}}{\partial t_\beta} = \frac{\partial (\rho_\beta \mathbf{v}_{z\beta})}{\partial t_\beta} = \rho_\beta \frac{\partial \mathbf{v}_{z\beta}}{\partial t_\beta} = \rho_\beta \mathbf{E}_{z\beta} \tag{80}$$

in accordance with classical results.

For the important case of propagation of electromagnetic waves in Casimir vacuum, equations (73) and (75) result in wave equations

$$\frac{\partial^2 \mathbf{E}_{z\beta}}{\partial t_\beta^2} = c_\beta^2 \nabla^2 \mathbf{E}_{z\beta} \tag{81}$$

$$\frac{\partial^2 \mathbf{H}_\beta}{\partial t_\beta^2} = c_\beta^2 \nabla^2 \mathbf{H}_\beta \tag{82}$$

Next, a third wave equation [37] will be associated with radial electric field intensity $\mathbf{E}_{r\beta}$ given by (78). Hence, for propagation of electromagnetic waves in Casimir vacuum one obtains wave equations

$$\partial^2 \boldsymbol{\phi}_\beta / \partial t_\beta^2 = c_\beta^2 \nabla^2 \boldsymbol{\phi}_\beta, \ \boldsymbol{\phi}_\beta = (\mathbf{E}_{z\beta}, \mathbf{H}_\beta, \mathbf{E}_{r\beta}) \tag{83}$$

and corresponding momentum waves as discussed in [37]

$$\partial^2 \boldsymbol{\zeta}_\beta / \partial t_\beta^2 = c_\beta^2 \nabla^2 \boldsymbol{\zeta}_\beta, \ \boldsymbol{\zeta}_\beta = (\mathbf{v}_{z\beta}, \boldsymbol{\omega}_\beta, \mathbf{v}_{r\beta}) \tag{84}$$

The three velocity waves (84) result in atomic energies due to translational, rotational, and radial harmonic motions given by

$$\varepsilon_{z\beta} = \int \mathbf{F}_{z\beta}.\mathrm{d}\mathbf{z}_\beta = \int \frac{\mathrm{d}\mathbf{p}_{z\beta}}{\mathrm{dt}_\beta}.\,\mathrm{d}\mathbf{z}_\beta = \\ = \mathrm{m}_\beta \mathbf{v}_{z\beta}^2/2 = \mathrm{m}_\beta \mathbf{v}_{+z\beta}^2 = \mathrm{k}T_\beta \tag{85}$$

$$\varepsilon_{\theta\beta} = \int \mathbf{F}_{\theta\beta}.\mathbf{d}\sigma = \int \frac{\mathbf{dp}_{\theta\beta}}{\mathrm{dt}_\beta}.\,\mathrm{rd}\boldsymbol{\theta}_\beta = \\ = \mathrm{m}_\beta \mathrm{r}^2 \boldsymbol{\omega}_{\theta\beta}^2/2 = \mathrm{I}_\beta \boldsymbol{\omega}_{+\theta\beta}^2 = \mathrm{k}T_\beta \tag{86}$$

$$\varepsilon_{r\beta} = \int \mathbf{F}_{r\beta}.\mathrm{d}\mathbf{r} = \int \chi_{r\beta}\mathbf{r}_\beta.\,\mathrm{d}\mathbf{r}_\beta = \\ = m_\beta \boldsymbol{\omega}_{r\beta}^2 \mathrm{r}_\beta^2/2 = \mathrm{m}_\beta \mathrm{v}_{+r\beta}^2 = \mathrm{k}T_\beta \tag{87}$$

For radial potential energy in (87) substitution from definition of spring constant $\chi_{r\beta} = \mathrm{m}_\beta \omega_{r\beta}^2$ has been made. Hence, the three energies (85)-(87) correspond to three fields $(\mathbf{E}_{z\beta}, \mathbf{H}_{\theta\beta}, \mathbf{E}_{r\beta})$.

At thermodynamic equilibrium, the energies due to harmonic oscillations in axial, angular, and radial coordinate directions give *atomic internal energy* or photon *electromagnetic mass* [34]

$$\hat{\mathrm{u}}_\beta = \hat{\varepsilon}_{EM} = \varepsilon_{z\beta} + \varepsilon_{\theta\beta} + \varepsilon_{r\beta} = 3\mathrm{k}T_\beta \tag{88}$$

The correction factor of 3/2 discussed by Bass and Schrödinger [139] is no longer encountered because the factor of ½ is taken care of by 2 coordinate directions mentioned above, and the factor 3 is accounted for by axial, radial, and angular degrees of freedoms.

Finally, photon peculiar velocity (3) results in *Poincaré stress* or pressure wave [37] giving photon *potential energy* or *gravitational mass* [36,37]

$$\varepsilon_{p\beta} = \hat{\varepsilon}_{GR} = \varepsilon_o \mathrm{E}_{+\mathrm{p}\beta}^2 = p_\beta \hat{\mathrm{v}} = \\ = m_\beta < V_{+\beta}'^2 > /3 = m_\beta < V_{+z\beta}'^2 > = \mathrm{k}T_\beta \tag{89}$$

At thermodynamic equilibrium, Boltzmann principle of equipartition of energy requires equality of energy of all four degrees of freedom [34]

$$\varepsilon_{z\beta} = \varepsilon_{\theta\beta} = \varepsilon_{r\beta} = \varepsilon_{p\beta} = \mathrm{k}T_\beta \tag{90}$$

Hence, total photon atomic energy or atomic enthalpy is

$$\hat{\varepsilon}_{\mathrm{total}} = \hat{\varepsilon}_{EM} + \hat{\varepsilon}_{GR} = \varepsilon_o \mathrm{E}_{+z\beta}^2 + \mu_o \mathrm{H}_{+\theta\beta}^2 + \\ + \varepsilon_o \mathrm{E}_{+r\beta}^2 + \varepsilon_o \mathrm{E}_{+p\beta}^2 = 4\mathrm{k}T_\beta \tag{91}$$

The result (91) is in agreement with the classical [57,126,135-137] total electromagnetic energy

$$\hat{\varepsilon}_{\mathrm{total}} = \varepsilon_o \mathrm{E}'^2/2 + \mu_o \mathrm{H}'^2/2 = \\ = \varepsilon_o \mathrm{E}_+'^2 + \mu_o \mathrm{H}_+'^2 = 2\mathrm{k}T_\beta' = 4\mathrm{k}T_\beta \tag{92}$$

since $T' = 2T$ as discussed in [34].

In summary, photons are assumed to have harmonic oscillations in axial (z_+, z_-), angular (θ_+, θ_-), and radial $(\mathrm{r}_+, \mathrm{r}_-)$ *coordinate directions* as schematically shown in Fig. 3,

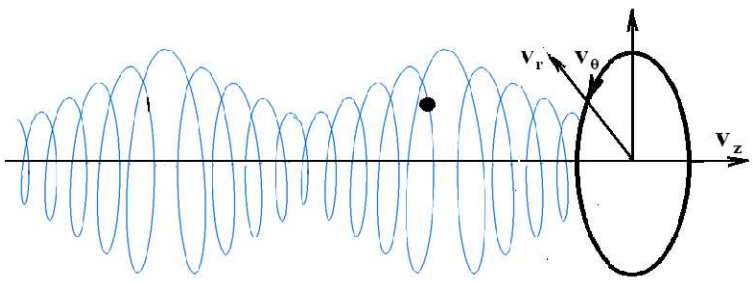


Figure 15 – Trajectory of photon "torpedo" with axial, angular, and radial harmonic velocities. The influence of random peculiar velocity is not shown.

Thus, we have four-coordinates $(z_\beta, \theta_\beta, r_\beta, z'_\beta)$ and corresponding four-velocity $(\mathbf{v}_{z\beta}, \mathbf{v}_{\theta\beta}, \mathbf{v}_{r\beta}, V'_{z\beta})$ and four-energy $(\varepsilon_{z\beta}, \varepsilon_{r\beta}, \varepsilon_{v\beta}, \varepsilon_{p\beta})$. The new hydrodynamic model has four fields $(\mathbf{E}_{z\beta}, \mathbf{H}_{\theta\beta}, \mathbf{E}_{r\beta}, \mathbf{E}_{p\beta})$ as opposed to the two classical electromagnetic fields $(\mathbf{E}', \mathbf{H}')$. The first field is new electric field and is called Lorentz *longitudinal electric field* $\mathbf{E}_{z\beta} = \mathbf{L}_{z\beta}$ associated with periodic compression and rarefaction of light wave. The last electric field $\mathbf{E}_{p\beta}$ is also new and corresponds to the wave function $\Psi_{z\beta}$ defined in (67) hence associated with the velocity potential of peculiar velocity $V'_{z\beta} = -\nabla\Phi'$ that is the hidden *pilot wave* of de Broglie-Bohm [3, 90, 106]. The four-waves associated with translational, rotational, vibrational, and "internal" degrees of freedom are schematically shown in Fig 16.

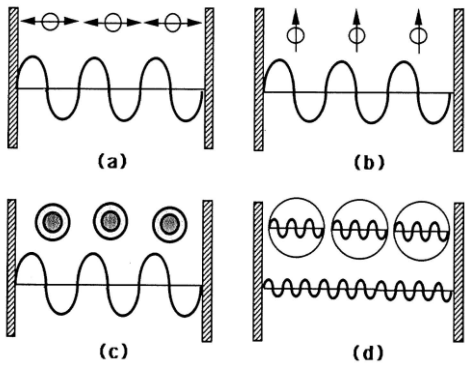


Figure 16 – Quantization of (a) translational (b) rotational (c) vibrational (d) internal waves of particles.

In Section 4, it was shown that particle peculiar velocity leads to *external pressure* that was identified as *Poincaré stress* responsible for particle stability [35-37]. The velocity potential of peculiar velocity is the imaginary part of quantum mechanics wave function (67). Since for stable particle, external pressure must be equal to internal pressure that accounts for the potential energy part (71) of Helmholtz decomposition of total thermal energy (84) identified as dark energy [33, 34]. Also, as discussed in [33], potential energy or dark matter of scale β is identified as the total energy (enthalpy) of the adjacent lower scale β–1. Finally, in the hierarchies of wave functions shown in (68), *internal wave function* of scale β decompactifies into four-wave functions of the next lower scale β–1 (see Fig. 7). Therefore, the internal wave function plays a more complex and significant role in the hierarchical model (Fig. 1) and hence cannot be explicitly revealed in Fig. 16.

The invariant hydrodynamic model described above is harmonious with Maxwell [57] and Lorentz [126] classical electrodynamics (72)-(75). Of course closer equivalence requires addition of a Coulomb scalar potential to account for electrostatic forces parallel to Newton gravitational potential. Also, in the hydrodynamic model, Lorentz [126] force on charged particles assumes the form

$$\mathbf{F}_\beta = \rho_\beta(\mathbf{E}_\beta + \mathbf{v}_\beta \times \mathbf{B}_\beta) = \rho_\beta(\mathbf{a}_\beta - 2\mathbf{v}_\beta \times \boldsymbol{\omega}_\beta) \quad (93)$$

with the two terms respectively representing linear and Coriolis accelerations, thus providing a reasonable hydrodynamic interpretation of Lorentz force. Therefore, the results presented in this Section suggest that future development of exact hydrodynamic model of Maxwell classical theory of electromagnetism is conceivable and in need of further investigations.

The primary obstacle in bridging the gap between hydrodynamics and electrodynamics concerns uncertainty about the exact nature of concept of electric *charge*. In view of the well-known similarity between Newton law of gravity and Coulomb law in electrostatics, it is natural to expect that the concept of charge is somehow related to mass. This is also supported by occurrence of charge in the expression of electric force when electric field strength $\mathbf{E}_\beta$ is identified as acceleration (78). It is most interesting that the numerical magnitude of the elementary charge of electron is almost exactly given by

$$e = \frac{1843 \times m_e \times c}{\pi} \approx$$
$$\approx \frac{1843 \times 9.10956 \times 10^{-31} \times 2.998 \times 10^{8}}{\pi} = \quad (94)$$
$$= 1.60215 \text{ C}$$

where the proton to electron mass ratio is based on photon mass in (28)

$$\frac{m_p}{m_e} \approx 1843 \quad (95)$$

If instead of (95), the classical ratio $m_p / m_e \approx 1836.2$ is employed in (94), one gets a smaller value $e \approx 1.59624 \times 10^{-19}$ C for elementary electron charge.

A second major difficulty to resolve is that as opposed to gravitation forces that are always attractive, electromagnetic forces could also be repulsive. Amongst many possibilities, the observed attractive and repulsive forces between "charged" particles could be related to the sense of their rotation in view of Biot-Savart law [137]

$$\mathbf{H} = \mathrm{I} d\boldsymbol{\ell} \times \hat{\mathbf{r}} / \mathrm{r}^2 \tag{96}$$

Also, because of the definition of magnetic field and its connection to vorticity by (77), the concept of charge is expected to be related to electron iso-spin defined in (8) as

$$\varpi_\beta = \nabla \times \mathbf{u}_\beta = \boldsymbol{\omega}_{\beta-1} \tag{97}$$

Indeed, spring constant that involves both mass and angular frequency

$$\chi_\beta = \boldsymbol{\omega}_\beta^2 \mathrm{m}_\beta \tag{98}$$

has been connected to electron charge by Jackson [136] and hence may also help in future understanding of exact physical nature of charge in physics.

In closing this section, it is instructive to note that the universality of transition between turbulent (highly dissipative) and laminar (weakly dissipative) flows across many scales (Fig. 1) discussed in a recent study [140], is in complete harmony with the hydrodynamic model of electrodynamics described above. Therefore, starting with the pioneering discovery of superconductivity by Onnes [141] at electrodynamic scale, one identifies super-luminosity (laser action) at much smaller scale of optics or dry hydrodynamics [17], super-fluidity at larger molecular dynamics scale, and laminar flow at much larger hydrodynamic scale. At the exceedingly large cosmic scales, besides the well-known cosmic lasers due to amplification of light waves, one anticipates much more powerful rays associated with stimulated amplification of tachyon waves. Hence, generalization of all such phenomena could be referred to as *matter-wave amplification due to stimulated emission of de Broglie matter wave packets*.

Implications to Quantum Cosmology, Everett Multiverse, and Quantum Gravity

An important advantage of the invariant model of Boltzmann statistical mechanics and the associated generalized thermodynamics (Fig. 1) is in helping to extrapolate knowledge from intermediate scales to much larger and much smaller scales that are less accessible to our ordinary physical intuition. As described in recent studies [90, 140], a factor of approximately 10^{17} separates the scales of five major statistical fields in our universe starting with tachyon (graviton)-dynamics at exceedingly small Planck scale 10^{-35} m, followed by electrodynamics 10^{-18} m, hydrodynamics 10^{0} m, astrophysics 10^{18} m, and finally galactic-dynamics (cosmology) 10^{35} m. Each statistical field has its "atomic" particle namely, graviton, electron, fluid-element, star, and galaxy. In the previous Section, the intuitively accessible hydrodynamics at intermediate scale was employed to model extremely small scale of electrodynamics. In the same spirit, in the present section, physical concepts from intermediate scale of classical hydrodynamics will be employed to model the exceedingly large scale of cosmology.

In view of the extremely large number of galaxies and large distances between them, flow of galaxies in cosmology [142-145] may be assumed as ideal gas thus follow invariant Boltzmann statistical mechanics and the associated generalized thermodynamics [140,146]. Hence, conservation equations (4)-(7) describe dynamics of spectrum of galactic clusters containing a corresponding spectrum of galaxies as "atoms" of *turbulent statistical field* $\mathbb{F}_\beta = \mathbb{F}_\mathrm{g}$ that resides within a *background space* $\mathbb{S}_\mathrm{g}$ defined as the field of astrophysics $\mathbb{S}_\mathrm{g} = \mathbb{F}_\mathrm{s}$ at the next lower scale of $\beta - 1 = \mathrm{s}$.

Ironically, turbulent hydrodynamic model of cosmology following conservation equations (4)-(7) is precisely the quantum theory of gravity as a *dissipative deterministic dynamic system* recommended by 't Hooft [147]. At cosmic scales, Navier-Stokes equation motion (19) and Helmholtz vorticity equation (20) account for dissipation of

ordered motions across hierarchy of scales into random thermal motions. Dissipations of ordered translational, rotational, and pulsational velocities into random thermal velocities are schematically shown in Fig. 17.

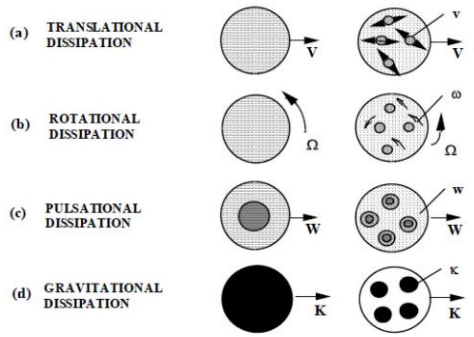


Figure 17 – Dissipation of momentum from global $(\mathbf{V}_z, \Omega_\theta, \mathbf{W}_r, \mathbf{K})$ to local $(\mathbf{v}_z, \omega_\theta, \mathbf{w}_r, \kappa)$: (a) translational (b) rotational (c) vibrational (d) gravitational degrees of freedom [33].

The last type of dissipation in Fig. 17 called *gravitational dissipation* [33, 147] corresponds to dissipation of large-scale curvature to random small-scale curvature such as a large droplet breaking up into many small droplets in sprays.

In equilibrium galactic-dynamics EGD field at scale $\beta = \mathbf{g}$, every "point" of cosmic space will be occupied by either "atom", i.e., a galaxy or vacuum [73]

$$\mathbb{S}_g = \begin{cases} \hat{\mathbf{m}}_g \equiv \mathbf{vacuum} = \mathbf{V}_g \\ \varnothing \equiv (\mathbf{vacuum} - \mathbf{vacuum}) = \mathbf{VV}_g = \mathbf{V}_s \end{cases} \tag{99}$$

Therefore, cosmic vacuum like Casimir vacuum shown in Fig. 3 is not empty and the model (Fig. 1) suggests a hierarchy of vacua defined as [33]

$$\begin{aligned} (\mathbf{vacuum} - \mathbf{vacuum})_\beta &= \mathbf{VV}_\beta = \\ &= (\mathbf{vacuum})_{\beta-1} = \mathbf{V}_{\beta-1} \end{aligned} \tag{100}$$

To emphasize the importance of such hierarchies of vacua to cosmology we make the following quotation from 't Hooft [148]

This means that there is one very special state where all energies are zero: the vacuum state. Identifying the vacuum state is particularly difficult in our theory, but it seems that the vacuum also poses problems in other approaches. In loop quantum gravity, it is notoriously difficult to say exactly what the vacuum state is in terms of the fundamental loop states that were introduced there. In superstring theory, there are many candidates for the vacuum, all being distinctly characterized by the boundary conditions and the fluxes present in the compactified par of space-time. String theory ends up leaving an entire 'landscape' of vacuum states with no further indication as to which of these to pick. It is of crucial importance in any theory of Planck length physics to identify and describe in detail the vacuum state.

It is emphasized that according to the present model of physical space (Fig. 3), the only true and absolute vacuum-vacuum is the *white hole* that is a singularity of the field with exact density $\rho_{WH} = \mathbf{0}$.

To describe hydrodynamics of the universe, following Section 3, the atomic (length, time, mass) of cosmic field are defined as the *most probable* (length, time, mass) of astrophysical statistical field

$$(\hat{\lambda}_g, \hat{\tau}_g, \hat{\mathbf{m}}_g) = (\lambda_{w,s}, \tau_{w,s}, \mathbf{m}_{w,s}) \tag{101}$$

At thermodynamic equilibrium, the equality of temperature of cosmic and astrophysical fields $T_g = T_s = \lambda_{w,s}$ gives the atomic mass unit of cosmic field $\hat{\mathbf{m}}_g = \mathbf{m}_{w,s}$. Also, from the definition of temperature $\mathbf{kT}_s = \mathbf{m}_{w,s}\mathbf{v}_{w,s}^2$ and most probable speed $\mathbf{v}_{w,m} = \lambda_{w,m} / \tau_{w,m}$, one obtains the atomic time $\hat{\mathbf{t}}_g = \tau_{w,m}$ of equilibrium galactic dynamics (EGD) field.

Having defined the atomic units of space, time, and mass, one asks the question that was asked by both Boltzmann and Planck namely: given the total number of atoms (galaxies) N and the total energy $H = 4N\mathrm{k}T$ of the universe, what is the distribution of sizes of galactic clusters N_j that results in *stochastically stationary* cosmic field. The probability of cluster of size N_j is identified as the *inverse* of what Boltzmann called *number of complexions* W_j that is given by Boltzmann-Planck formula [34]

$$P_j = \frac{N_j!\,(g_j - 1)!}{(N_j + g_j - 1)!} = \frac{1}{W_j}, \quad P = \prod_j P_j = \frac{1}{W} \tag{102}$$

Following Planck [100], equation (102) leads to invariant distribution for energy spectra of galaxies.

$$\frac{\varepsilon_g dN_g}{V} = \frac{8\pi h}{u_g^3}\frac{\nu_g^3}{e^{h\nu_g/kT}-1}d\nu_g \qquad (103)$$

At thermodynamic equilibrium between matter and radiation, equation (103) corresponds to Planck [100] spectral energy distribution of equilibrium radiation with ¾ and ¼ fractions of total energy on $\nu_j > \nu_w$ and $\nu_j < \nu_w$ sides of Wien frequency ν_w as shown in Fig. 18.

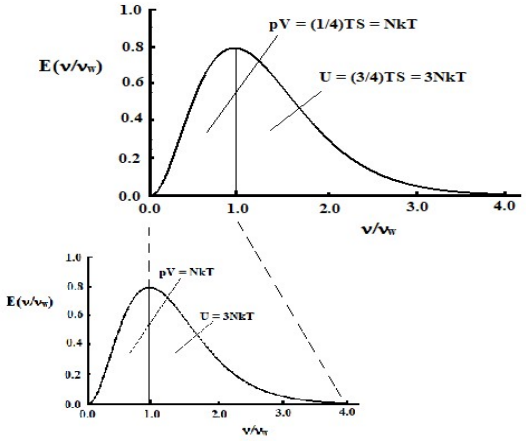


Figure 18 – Re-normalized Planck energy distributions as a function of ν / ν_w [36].

Comparison of astrophysical measurements [149] with Planck equilibrium energy spectrum at approximate temperature 2.73 m is shown in Fig. 19.

The fluctuations of the observed temperature perturbations in Fig. 19 corresponding to Penzias-Wilson [121,122,123] cosmic microwave background temperature $T_{CMB} \approx 2.73\ \mathrm{m}$ is known to be about $\nabla T \approx 1/10{,}000\ \tilde{\mathrm{m}}$ Normalizing data in Fig. 19 with Wien frequency $\nu / \nu_w = \nu / 225$, leads to $\nu_\infty / \nu_w \approx 900/225 = 4$ as the limit beyond which the spectral energy vanishes in close agreement with normalized Planck distribution in Fig. 18.

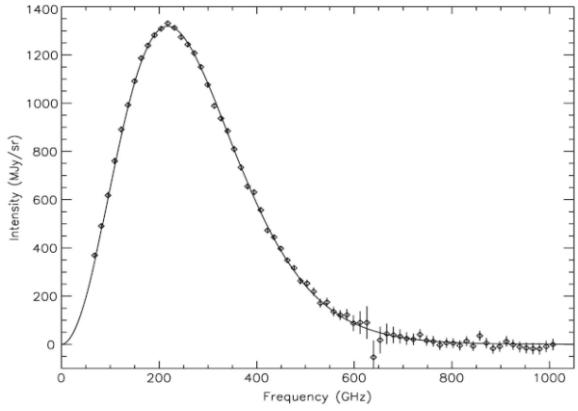


Figure 19 – Mean spectrums associated with the velocity of the solar system with respect to the CMB. The line is the a priori prediction based on the *WMAP* velocity and the previous FIRAS calibration. The uncertainties are the noise from the FIRAS measurements. The error bars are slightly misleading, because they do not show the correlations, but the correlated errors are properly treated in the fit. [149].

Close agreement of cosmic microwave background temperature with Planck distribution (Fig. 19) is strong evidence for near equilibrium state of universe. Therefore, it is natural to expect that distribution of speed of galaxies should follow invariant normalized Maxwell-Boltzmann distribution [36]

$$\frac{dN_{u\beta}}{N} = 4\pi \left(\frac{m_\beta}{2\pi k T_\beta}\right)^{3/2} u_\beta^2 \, e^{-m_\beta u_\beta^2/2kT_\beta} du_\beta \quad (104)$$

Calculated Normalized Maxwell-Boltzmann (NMB) distribution as a function of speed re-normalized with respect to the most-probable or Wien speed $v / v_w = \lambda_w / \lambda$ at two adjacent scales are shown in Fig. 20.

NMB distribution in Fig. 20 closely agrees with typical observed distribution of fractions of galaxy clusters containing *N* galaxies reported by Saslaw and Crane [150], Lahav and Saslaw [151], and Saslaw [152] shown in Figs. 21 and 22.

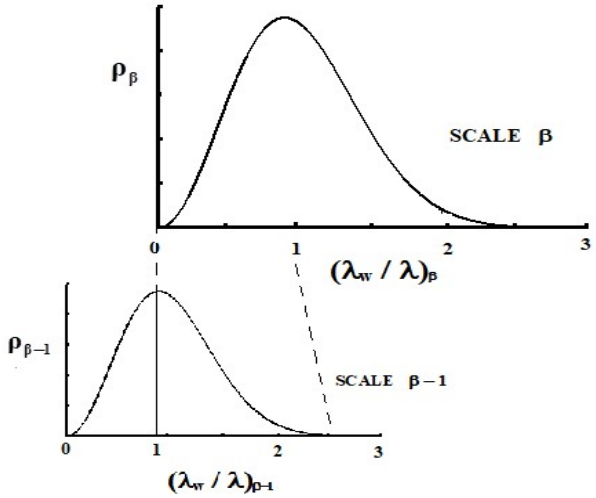


Figure 20 – Re-normalized Maxwell-Boltzmann distribution as a function of dimensionless speed $v / v_w = \lambda_w / \lambda$ [36].

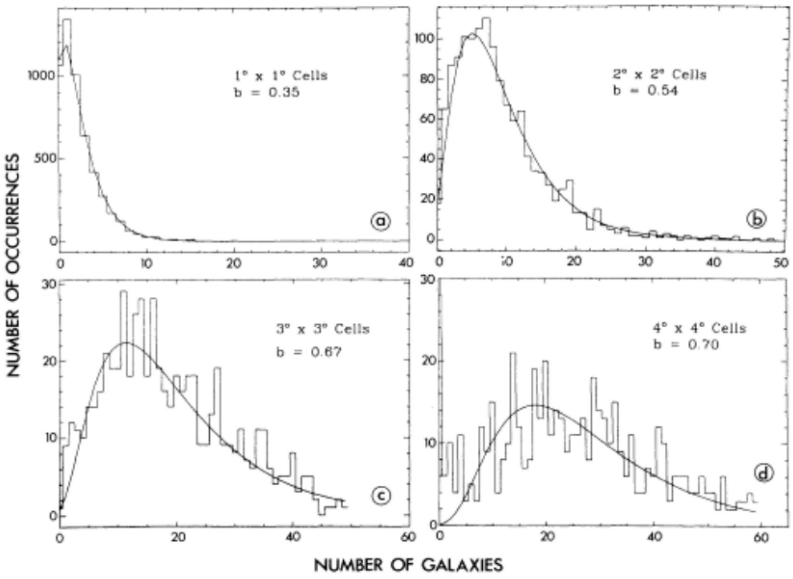


Figure 21 – Observed distribution fraction of galaxy clusters containing N galaxies [150].

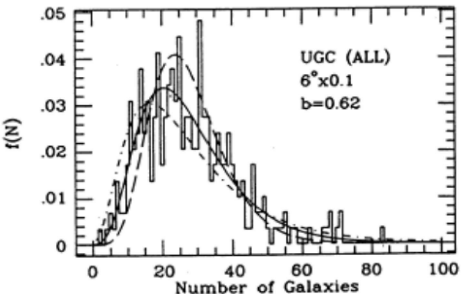


Figure 22 – Observed distribution fraction of galaxy clusters containing N galaxies [151].

Normalizing the data in Figs. 21d or 22 with Wien number $N_w = 20$, leads to maximum $N_\infty / N_w \approx 60/20 = 3$ in agreement with NMB distribution in Fig. 20.

Planck distribution of cosmic microwave background radiation energy in Figs. 18 and 19, Maxwell-Boltzmann distribution of galactic sizes hence speeds in Figs. 20-22, derivation of invariant Schrödinger equation from invariant Bernoulli equation [37], and finally the observed *quantum jumps* in red shifts of galaxies [153] are all in harmony with the hypothesis of quantum cosmology. Assuming incompressible potential flow, dynamics of universe will be governed by invariant Schrödinger equation with peculiar velocity of galaxies defining its wave function. However, a more realistic model will be the *dissipative deterministic dynamic system* theory of quantum gravity proposed by 't Hooft [147], involving solutions of conservation equations (4)-(7) under relativistic (compressible) and viscous hydrodynamics. In view of invariant Schrödinger (65)-(67) and Dirac relativistic wave equation (71), the present invariant hydrodynamic model may help in closure of the gap between quantum theory of gravitation and quantum mechanics. The connection of quantum mechanics with Einstein [154] GTR is less intuitive due to the complex mathematical structure of tensors and differential geometry of Riemann.

Interestingly, in a recent excellent investigation by Nugayev [155], concerning the historical development of GTR, research programs of Nordström *scalar theory*, Abraham *vector theory*, and Einstein *tensor theory* of gravitation were described. Clearly, the hydrodynamic model of cosmology involving (4)-(7) contain primarily scalar and vector terms, and only one tensor namely total stress tensor P_{ij} in (6). Therefore, it is expected that scalar and vector theories of Nordström and Abraham discussed in [155] will play important roles in future development of quantum theories of gravitation. This is in part because scientific developments also follow Hamiltonian principle of least energy, and following more familiar classical hydrodynamics with scalar and vector theories are much simpler and intuitively accessible than complex tensor theories of Riemannian geometry!

When physical space is identified as a fluid, it is natural to attribute gravitational forces to gradients of ether density hence pressure as schematically shown in Fig. 23.

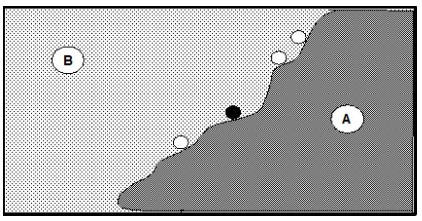


Figure 23 – Schematic diagram of motion in a "curved space". The particle moves along the trajectory defined as the interface between higher density medium A and lower density medium B.

Hence, in a recent investigation [140], gravitational force was related to normal stress expressed as diffusional flux of momentum applying (11)

$$\tau_{iig} = \frac{F_g}{A} = \rho_e v_{ie} V_{iie} = \rho_e v_{ie}(-D_e \nabla \rho_e / \rho_e) \quad (105)$$

leading to Newton law of gravitation as pressure gradient of ether or Casimir [63] vacuum [140, 156]

$$F_g = -m_e \nabla p_e / \rho_e = m_e g \quad (106)$$

with gravitational acceleration defined as

$$g = -\nabla p_e / \rho_e \quad (107)$$

According to (105)-(107), solution of conservation equations (4)-(7) under appropriate boundary conditions provide (ρ, T, p) scalar fields hence gravitational forces at each location. Following Poincaré [82] description of hyperbolic

geometry and Gauss classical definition of intrinsic surface curvature, a one-dimensional line curvature was recently defined as atomic line density [140] in accordance with (64)

$$\kappa_x \equiv \frac{N_x}{L_x} = \frac{N_x}{N_x \lambda_w} = \frac{1}{\lambda_w} = \frac{1}{T} \qquad (108)$$

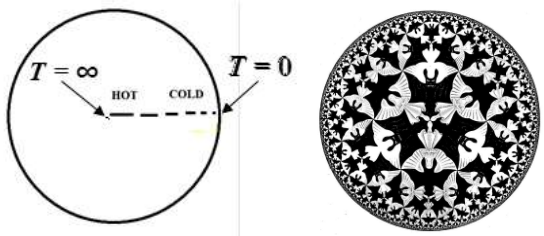
Also, since for ideal gas $p_\beta = \rho_\beta R_\beta T_\beta$, density and temperature are inversely related, by (109) local scaler curvature is defined as deviation of density from that of equilibrium Casimir vacuum [37,140]

$$\kappa = \rho - \rho_v \qquad (109)$$

Hence, cosmic field involves three geometries

$$\begin{array}{l} \kappa > 0 \text{ Matter (Riemannian space)} \\ \kappa = 0 \text{ Casimir Vacuum (Euclidean space)} \\ \kappa < 0 \text{ Anti-matter (Lobachevskian space)} \end{array} \qquad (110)$$

In his description of hyperbolic geometry, Poincaré [82] introduced what is known as *Poincaré disk* schematically shown in Fig. 24.



(a) (b)

Figure 24 – (a) Poincare disk with white hole at origin $T \to \infty$ and black hole on circumference $T \to 0$ **(b)** Circle Limit IV (Heaven and hell) by M.C. Escher [74].

Also shown in Fig, 24, is the wonderful woodcut print by Escher [157] called Circle Limit IV (heaven and hell) graphically showing hyperbolic geometry. The one-dimensional line density as curvature (108) hence (109) may be viewed as *generalized non-Euclidean geometry* that is in harmony with perceptions of Poincaré [82] as well as non-standard analysis discussed in [158].

The temperature limits $(T \to \infty, T \to 0)$ correspond to (white hole, black hole) at the (origin, circumference) of Poincaré disk. Concerning the temperature limits in Fig. 24, it is interesting to examine Maxwell-Boltzmann distribution in Fig. 5 for ether or Casimir vacuum (Fig. 3) at three different thermodynamic temperatures as schematically shown in Fig. 25. One notes that as temperature is increased, eventually all photons as Bose-Einstein liquid evaporate into gravitons and the entire distribution function collapses on the horizontal axis that is identified as *white hole* in accordance with Figs. 3 and 24. On the other hand, as the temperature is steadily decreased, eventually all space, i.e., superfluid Bose-Einstein condensate solidify and all distribution curves shown in Fig. 25 "freeze" by collapsing on the vertical axis corresponding to *solid light* or "black hole" [37,90] singularity in accordance with Figs. 3 and 24.

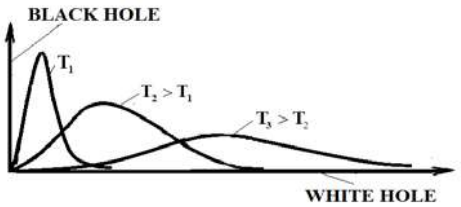


Figure 25 – Black-hole and White-hole limits on maxwell-Boltzmann speed distribution as temperature of ether or Casimir vacuum approaches zero and infinity.

Because of the change of units $(\mathrm{cm/m} \to 1/100)$ required by Wien displacement law $\lambda_w T = 0.29 \ \mathrm{cm.K} = 0.0029 \ \mathrm{m^2}$ due to modified dimension of absolute thermodynamic temperature [34], the classical formula for transformation to Kelvin absolute temperature becomes

$$T(\mathrm{m}) = {}^{\circ}C(\mathrm{m}) + 2.731(\mathrm{m}) \qquad (111)$$

The constant 2.731 in (111) is close to Penzias-Wilson [121-123] cosmic microwave background temperature $T_{CMB} \approx 2.73 \ \mathrm{m}$ suggesting that the entire universe is at ice temperature. The fact that our universe is very close to the state of thermodynamic equilibrium is evidenced by the observed thermal nature of Penzias-Wilson [121-123] cosmic background temperature $T_{CMB} \approx 2.73 \ \mathrm{m}$ (Fig. 19), as well as equilibrium distribution of speed of galaxies shown in Figs. 21 and 22.

As discussed in [140], the magnitude of Planck temperature

$$T_p = (hc^5/Gk^2)^{1/2} \approx 3.55\times10^{32}\ \text{m} \qquad (112)$$

introduces a paradox since it corresponds to most probable wavelength of photon thermal oscillation that is larger than the reported size of our universe 10^{26}. In view of Fig. 1, this suggests that our universe with (atom, element, system) lengths

$$\text{EGD} \quad (\ell_g,\ \lambda_g,\ L_g) = (10^{18},\ 10^{26},\ 10^{35})\ \text{m} \qquad (113)$$

is Lemaître [159] "*primordial atom*" of a much larger Everett [160] *multiverse* with (atom, element, system) lengths [140]

$$\text{EUD} \quad (\ell_u,\ \lambda_u,\ L_u) = (10^{35},\ 10^{43},\ 10^{52})\ \text{m} \qquad (114)$$

This is in harmony with hierarchy shown in Fig. 1 as well as inflationary theories of cosmology [161-164].

Interestingly, recent cosmological observations have revealed a small asymmetry in the power spectrum from the right versus the left side of our universe [165] schematically shown in Fig. 26.

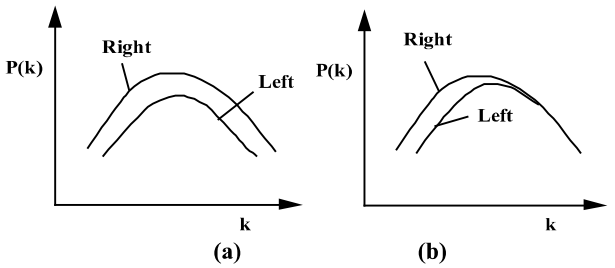


Figure 26 – Asymmetry in measured cosmic power spectrum (a) calculated (b) measured [165].

It is reasonable to attribute the asymmetry in Fig. 26 to the fact that our universe is rotating [35]. When our universe is identified as Lemaître [159] "*primordial atom*" of a much larger Everett [160] *multiverse*, finite "iso-spin" of our universe within multiverse becomes understandable and harmonious with the observed rotation of almost all galaxies. Moreover, this is also in harmony with Gödel [166] wonderful 1949 study introducing *closed time-like solutions* of Einstein field equations in rotating universe with non-vanishing cosmological constant. Furthermore, Gödel [166] rotating universe is in harmony with Kerr [167] solution for rotating black hole.

In view of the scale-invariant nature of quantum mechanics (65)-(67), it is interesting to view the spectral sizes of ether or space "quanta" at thermodynamic equilibrium in "empty" de Sitter [168] universe given by Maxwell-Boltzmann distribution [140]. At any temperature, Poincaré *thermal measure* $T_\beta = \lambda_{w\beta}$ [90] could be applied to define respectively microscopic (intensive) *most probable atomic* volume $\hat{v}_{w\beta}$ and macroscopic (extensive) *system* volume V_β as

$$\hat{v}_{w\beta} = \lambda_{w\beta}^3 = T_\beta^3\ ,\quad V_\beta = \sum \hat{v}_{w\beta} = N_\beta \hat{v}_{w\beta} \qquad (115)$$

Similarly, one defines a *spectrum* of *atomic* and *cluster* or "element" volumes

$$\hat{v}_{j\beta} = \lambda_{j\beta}^3\ ,\quad V_{j\beta} = \sum_i \hat{v}_{ij\beta} = N_{i\beta}\hat{v}_{ij\beta} \qquad (116)$$

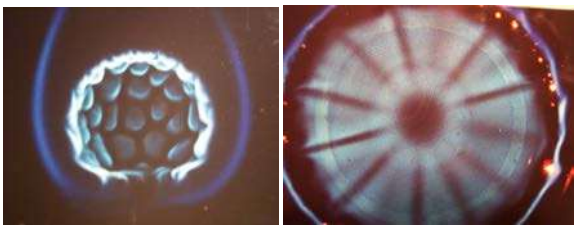
such that the total system volume in (115) can also be expressed as

$$V_\beta = \sum_j V_{j\beta} \qquad (117)$$

Therefore, according to the model (Fig. 1), physical space is composed of a spectrum of *element volumes* that contain corresponding spectrum of *atomic volumes* at thermodynamic equilibrium in harmony with modern concepts of loop quantum gravity [147,169-175]. Clearly, transformation of Wheeler-DeWitt equation [169-171] into Schrödinger equation (65)-(67) through resurrection of *internal* thermodynamic time [140], will help the closure of the gap between quantum gravity and quantum mechanics. A most fundamental aspects of quantum gravity is the critical role of probability due to stochastic nature of Casimir vacuum hence universality of chance [112,176-178] as an indispensable feature of quantum mechanics in harmony with perceptions of Heisenberg [75,79].

An outstanding problem of cosmology [142-145] is the origin of initial perturbations responsible for galaxy formation. Since universe is a chemically reactive system [179], in view of the scale invariance of the model (4)-(7), it is reasonable to expect that, similar to combustion

science at molecular-dynamic scale [40], thermos-diffusive instabilities govern hydrodynamics of the universe after explosion of Lemaître [159] "primordial atom" or Big Bang. Two examples of thermos-diffusive instability leading to cellular flames in rich premixed butane-air are shown in Fig. 27.



(a) (b)

Figure 27 – Formation of cellular thermo-diffusive instability in rich butane-air premixed flame with Lewis number based of deficient species Le < 1 stabilized on (a) A porous sphere (b) Stagnation-point flow against planer quartz plate.

The geometry of cellular flame depends on boundary conditions as seen in Fig. 27. For example, in 2-dimensional stagnation point flow as opposed axi-symmetric one shown in Fig. 27b, cellular flame instability leads to a row of parallel flame stripes. Relative scale-invariant diffusivities [36] of mass D_β, heat $\alpha_\beta = \sigma_\beta / \rho c_p$, and momentum $\tilde{\kappa}_\beta = \eta_\beta / \rho_\beta$ in (4)-(7) govern "flame" instabilities and their interactions according to hydro-thermo-diffusive theory of laminar flames [180].

In closing this Section, it is instructive to briefly consider another interesting and outstanding problem of cosmology namely the lack of knowledge of the boundary conditions of universe. Of course, without knowing such boundary conditions, the global hydrodynamics of universe cannot be determined from solution of (4)-(7). According to quantum mechanics (65-67), stability of particle is due to an external pressure known as Poincaré stress [36,37]. It is therefore reasonable to attribute stability of galactic clusters to a surrounding halo due to chaotic random motion of galaxies. Similarly, halos due to chaotic random motion of stars could be responsible for stability of galaxies.

Although the present thermodynamic model of cosmology cannot resolve the boundary condition problem, perhaps the following analogy will help in visualization of the problem. A classical problem of boundary conditions in fluid mechanics is known as *Stokes paradox* and concerns solution of equation of motion for creeping uniform laminar flow of viscous fluid across a rigid cylinder. No solutions that simultaneously satisfy vanishing velocity on the rigid cylinder and uniform velocity far away from cylinder could be found. However, recently Stokes paradox was resolved by finding the solution of modified form of equation of motion (19) for viscous flow across a rigid cylinder given by stream function [32]

$$\Psi_{RC} = -\xi(1 - 2/\xi + 1/\xi^2)\sin\theta \qquad (118)$$

where $\Psi_{RC} = \Psi'_{RC} / R'_i U$, $\xi = r'/R'_i$, and R'_i is radius of rigid cylinder. Some streamlines calculated from (118) are shown in Fig. 28.

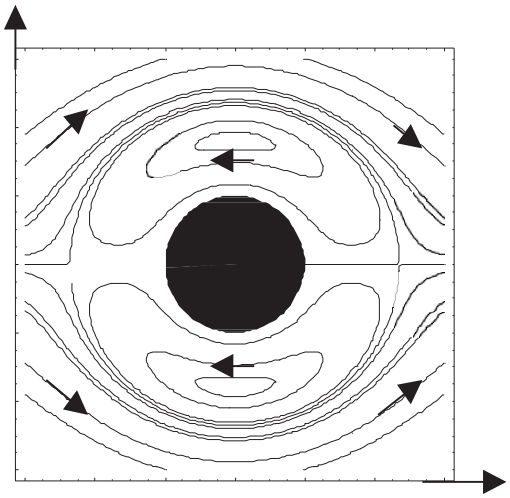


Figure 28 – Streamlines for Stokes uniform viscous flow across a rigid cylinder [32].

The radial and angular velocities obtained from (118) are

$$v_r = -(1 - 2/\xi + 1/\xi^2)\cos\theta\,, \qquad v_\theta = (1 - 1/\xi^2)\sin\theta \qquad (119)$$

As seen in Fig. 28, the problem of matching of solution to have vanishing velocities at the wall and uniform flow in far field is resolved through formation of two cylindrical line-vortices forming a closed viscous *boundary layer* with flow recirculation that in effect isolates the rigid cylinder from the outer potential flow. Thus, Stokes paradox helps to illustrate that nature in its infinite possibilities can accommodate diverse boundary

conditions through creation of intermediary transition layers such as the recirculation zone in Fig. 28. Although the boundary conditions of our universe remain to be determined by future investigations, it is reasonable to anticipate that like galaxies and cluster of galaxies, the boundary condition of our universe within much larger Everett [160] multiverse is also chaotic in nature [78,181].

Concluding Remarks

The study was focused on some implications of a scale-invariant model of Boltzmann statistical mechanic to four different areas pertaining to fundamental problems of theoretical physics. First part concerned quantization of space, time, and mass leading to clarification of concept of internal time, Rovelli [70] thermal time, invariant definition of atomic mass, mass-energy equivalence, and Joule-Mayer mechanical equivalent of heat. In the second part the model was applied to physical foundations of invariant Schrödinger equation of quantum mechanics as well as special theory of relativity. In particular, new perspectives concerning resolution of seven important problems regarding physical interpretation of quantum mechanics were presented.

In the third part of study the implication of the model to Maxwell theory of electromagnetism was studied. A hydrodynamic model of electromagnetism was introduced leading to new perspectives concerning the nature of electric and magnetic fields as well as that of electric charge. Finally, in part four of the study, some of the implications of the model to the fields of cosmology, quantum gravity, and quantum cosmology were addressed. The predicted equilibrium speed distribution of galaxies was found to be in good agreement with existing cosmological data. Physical space or Casimir vacuum was found to be composed of a spectrum of quantized space volume elements each composed of corresponding spectrum of atomic space volume quanta in harmony with modern theories of loop quantum gravity. The results were found to be in accordance with quantum gravity as a dissipative deterministic dynamic system proposed by ‘t Hooft [147].

Acknowledgments

This research was in part supported by NASA grant No. NAG3-1863.

References

1. L. de Broglie, Interference and corpuscular light, Nature 118, 441-442 (1926); Sur la possibilité de relier les phénomènes d'Interférence et de diffraction à la théorie des quanta de lumière, C. R. Acad. Sci. Paris, 183, 447-448 (1927); La structure atomique de la matière et du rayonnement et la mécanique ondulatoire, 184, 273-274 (1927); Sur le rôle des ondes continues en mécanique ondulatoire, 185, 380-382 (1927).
2. L. de Broglie, *Non-Linear Wave Mechanics: A Causal Interpretation*, Elsevier, New York, 1960.
3. L. de Broglie, The reinterpretation of wave mechanics, Found. Phys. 1(5), 5-15 (1970).
4. E. Madelung, Quantentheorie in hydrodynamischer form, Z. Physik. 40, 332-326 (1926).
5. E. Schrödinger, Über die Umkehrung der Naturgesetze, Sitzber Preuss Akad Wiss Phys-Math Kl 193, 144-153 (1931).
6. R. Fürth, Über Einige Beziehungen zwischen klassischer Staristik und Quantenmechanik, *Z.* Phys. 81, 143-162 (1933).
7. D. Bohm, A suggested interpretation of the quantum theory in terms of “Hidden Variables" I, Phys. Rev. 85(2), 166-179 (1952).
8. T. Takabayasi, On the foundation of quantum mechanics associated with classical pictures, Prog. Theor. Phys. 8(2), 143-182 (1952).
9. D. Bohm, and J. P. Vigier, Model of the causal interpretation of quantum theory in terms of a fluid with irregular fluctuations, Phys. Rev. 96 (1), 208-217 (1954).
10. E. Nelson, Derivation of the Schrödinger equation from Newtonian mechanics, Phys. Rev. 150 (4), 1079-1085 (1966).
11. E. Nelson, *Quantum Fluctuations*, Princeton University Press, Princeton, NJ,1 985.
12. L. de la Peña, New Foundation of Stochastic Theory of Quantum Mechanics, J. Math. Phys. 10(9), 1620-1630 (1969).
13. L. de la Peña, and A. M. Cetto, Does quantum mechanics accept a stochastic support? Found. Phys. 12(10), 1017-1037 (1982).
14. A. O. Barut, Schrödinger’s interpretation of □ as a continuous charge distribution, Ann. der Phys. 7(4-5), 31-36 (1988).
15. A. O. Barut, and A. J. Bracken, Zitterbewegung and the internal geometry of the electron, Phys. Rev. D 23 (10), 2454-2463 (1981).
16. J. P. Vigier, De Broglie waves on Dirac aether: A testable experimental assumption, *Lett. Nuvo Cim*. 29(14), 467-475 (1980); C. Cufaro Petroni, and J. P. Vigier, Dirac’s aether in relativistic quantum mechanics, Found. Phys. 13(2), 253-286 (1983);

J. P. Vigier, Derivation of inertia forces from the Einstein-de Broglie-Bohm (E.d.B.B) causal stochastic interpretation of quantum mechanics, Found. Phys. 25(10), 1461-1494 (1995).

17. F. T. Arecchi, and R. G. Harrison, *Instabilities and Chaos in Quantum Optics*, Springer-Verlag, Berlin, 1987.

18. O. Reynolds, On the dynamical theory of incompressible viscous fluid and the determination of the criterion, Phil. Trans. Roy. Soc. A 186(1), 123-164 (1895).

19. D. Enskog, Kinetische Theorie der Vorgange in Massig Verdunnten Gasen, Almqvist and Wiksells Boktryckeri-A.B., Uppsala, 1917. English translation: G. S. Brush, *Kinetic Theory*, 125-225, Pergamon press, New York, 1965.

20. G. I. Taylor, Statistical theory of turbulence-parts I-IV, Proc. Roy. Soc. A 151(873), 421-478 (1935).

21. T. Kármán, and L. Howarth, On the statistical theory of isotropic turbulence, Proc. Roy. Soc. A 164(917), 192-215 (1938).

22. H. P. Robertson, The Invariant theory of isotropic turbulence, Proc. Camb. Phil. Soc. 36, 209-223 (1940).

23. A. N. Kolmogorov, Local structure on turbulence in incompressible fluid, C. R. Acad. Sci. U. R. S. S. 30, 301-305 (1941); A refinement of previous hypothesis concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number, J. Fluid Mech. 13, 82-85 (1962).

24. A. M. Obukhov, On the distribution of energy in the spectrum of turbulent flow, C. R. Acad. Sci. U. R. S. S. 32, 19-22 (1941); Some specific features of atmospheric turbulence, J. Fluid *Mech.* 13, 77-81 (1962).

25. S. Chandrasekhar, Stochastic problems in physics and astronomy, Rev. Mod. Phys. 15(1), 1-89 (1943).

26. S. Chandrasekhar, *Stochastic, Statistical, and Hydrodynamic Problems in Physics and Astronomy*, Selected Papers, vol.3, University of Chicago Press, Chicago, 199-206, 1989.

27. W. Heisenberg, On the theory of statistical and isotropic turbulence, Proc. Roy. Soc. A 195, 402-406 (1948); Zur Statistischen Theorie der Turbulenz, Z. Phys. 124(7-12), 628-657 (1948).

28. G. K. Batchelor, *The Theory of Homogeneous Turbulence*, Cambridge University Press, Cambridge, 1953.

29. L. D. Landau, and E. M. Lifshitz, *Fluid Dynamics*, Pergamon Press, New York, 1959.

30. H. Tennekes, and J. L. Lumley, *A First Course in Turbulence*, MIT Press, 1972.

31. S. H. Sohrab, Invariant forms of conservation equations and some examples of their exact solutions, J. Energy Resources Technol. 136, 1-9 (2014).

32. S. H. Sohrab, Solutions of modified equation of motion for laminar flow across (within) rigid (liquid) and sphere and cylinder and resolution of Stokes paradox, *AIP Conference Proceedings* 1896, 130004 (2017).

33. S. H. Sohrab, Some implications of a scale invariant model of statistical mechanics to classical and relativistic thermodynamics, Int. J. Thermo. 17(4), 233-248 (2014).

34. S. H. Sohrab, On a scale-invariant model of statistical mechanics and the laws of thermodynamics J. Energy Resources and Technol. 138(3), 1-12 (2016).

35. S. H. Sohrab, Quantum theory of fields from Planck to cosmic scales, WSEAS Trans. Math. 9, 734-756 (2010).

36. S. H. Sohrab, On a scale invariant model of statistical mechanics, kinetic theory of ideal gas, and Riemann hypothesis, Int. J. Mod. Communication Tech. & Research. 3(6), 7-37 (2015).

37. S. H. Sohrab, Connecting Bernoulli and Schrödinger Equations and its Impact on Quantum-Mechanic Wave Function and Entanglement Problems, C.H. Skiadas and Y. Dimotikalis (eds.), *13th Chaotic Modeling and Simulation international Conference,* Springer Proceedings in Complexity, 2021.

38. R. S. de Groot, and P. Mazur, *Nonequilibrium Thermodynamics*, North-Holland, 1962.

39. H. Schlichting, *Boundary-Layer Theory*, McGraw Hill, New York, 1968.

40. F. A. Williams, *Combustion Theory*, 2nd Ed., Addison Wesley, New York, 1985.

41. J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird, *Molecular Theory of Gases and Liquids,* Wiley, New York, 1954.

42. S. Chapman, and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases*, Cambridge University Press, Cambridge, 1953.

43. R. Clausius, “Uber die Art der Bewegung, welche wir W€arme nennen,” Ann. Phys., 176(3), pp. 353–380 (1857).

44. R. Clausius, Ueber einen auf die Warme anwendbaren mechanischen Satz, Sitzungsberichte der Niedderrheinischen Gesellschaft, Bonn, Germany, 114–119 (1870).

45. J. H. Lugt, *Vortex Flow in Nature and Technology*, Wiley, New York, 1983.

46. O. Darrigol, Between hydrodynamics and elasticity theory: The first five births of the Navier-Stokes equation, *Arch. Hist. Exact Sci.* 56, 95-150, 2002.

47. A. B. Bahatia, and R. N. Singh, *Mechanics of Deformable Media*, IOP publication, Boston, 153, 1986.

48. O. Darrigol, *World of Flow,* Oxford University Press, New York, 123, 2005.

49. H. B. G. Onsager, Reciprocal relations in irreversible processes, I., *Phys. Rev.* 37, 405-425 (1931).

50. C, W, Oseen, Stokes formula and a related theorem in hydrodynamics, Ark. Mat. Astr. Fys. 6, 20. (1910).

51. G. Carrier, On slow viscous flow, Nonr-653-00-1 (1953).

52. H. von Helmholtz, Uber Integrale der hydrodynamischen Gleichungen, welche der Wirbelbewegung entsprechen, J. f˙ur die reine und angewandte Math-ematik, 55, 25-55, (1858).

53. Aristotle, in: *Time,* Westphal J. and Levenson, C., (Eds.), Hackett Publishing Company, Indianapolis, Indiana, 1993.

54. C. Huygens, *Treatise on Light*, p.14, Dover, 1912.

55. I. Newton, *Optics;* Dover: New York, 1952.

56. L. Euler, Réflexions sur l'espace et le temps. *Histoire de l'Academie Royale des sciences et belles lettres* 4, 324–333 (1748).

57. J. C. Maxwell, *A Treatise on Electricity and Magnetism*, Volumes 1 & 2, Dover Publication, New York, 1891.

58. J. Knox, Hendrik Antoon Lorentz, the Ether, and the General Theory of Relativity. In: *Einstein and the History of General Relativity*, Howard, D., and Stachel, J. (Eds.), 201-212, Birkhauser, Boston, 1989.

59. H. Poincaré, La théorie de Lorentz et le principe de réaction. Arch. Neerland. 5, 252-278 (1900); http://www.physicsinsights.org/poincare-1900.pdf

60. H. Poincaré, Sur la dynamique de l'électron, C. R. Acad. Sci., Paris 140, (1905).

61. H. Poincaré, Sur la dynamique de l'electron, Rend. Circ. Mat. Palermo 21(12), 9-175 (1906).

62. A. A. Logunov, *On the Articles by Henri Poincaré: "On the Dynamics of the Electron"*, Dubna: JINR, 2001.

63. H. B. G. Casimir, On the attraction between two perfectly conducting plates, Proc. K. Ned. Akad. Wet. 51, 793-795 (1948).

64. P. A. M. Dirac, Is there an aether?; Nature 168, 906 (1951).

65. E.T. Whittaker, *A History of the Theories of Aether and Electricity*, Vols. 1 &2, Tomash Publishers, New York, 1987.

66. H. A. Lorentz, Aether Theories and Aether Models, in: *Selected Works of L.H. Lorentz,* vol.5, Nersessian, N. J., and Cohen, H. F., (Eds.), Palm Publications, Nieuwerkerk, 1987, pp. 7-12.

67. G. Leibniz, Time is a Relation, In: *Time*, I. Westphal and C. Levenson, (Eds.), Hackett Publishing Company, Indianapolis, Indiana, p. 45, 1993.

68. W. Pauli, *Pauli Lectures on Physics,* Vol. 3, MIT Press, New York, 1973.

69. C. Rovelli, *The Order of Time*, Riverhead Books, 2018.

70. C. Rovelli, Statistical mechanics of gravity and the thermodynamical origin of time, Class. Quantum Grav. 10, 1549 (1993).

71 C. Rovelli, General relativistic statistical mechanics. *General Relativity and Quantum Cosmology*, 10.1103, Phys. Rev. D 87.084055, arXiv:1209.0065v2 (2012).

72. S. Hawking, *A Brief History of Time*, Bantam Book, New York, 1988.

73. Y. Vladimirov, N. Mitskievich, and J. Horsky, *Space Time Gravitation*, Mir Publishers, Moscow, 1987.

74. S. H. Sohrab, An invariant model of Boltzmann statistical mechanics and some of its implications to thermodynamics and quantum nature of space and time, WSEAS Tran, Appl. Theo. Mech. 13, 199-212 (2018).

75. W. Heisenberg, *The Physical Principles of Quantum Theory,* Dover, New York,1949.

76. A. Connes, On the Fine Structure of Spacetime. In: *On Space and Time*, Shahn M., ed.; Cambridge University Press, 2008, pp. 196-237.

77. H. Minkowski, Space and Time, in *Theory of Relativity*, p. 75, Dover, New York, 1952.

78. F. Verhulst, *Henri Poincaré, Impatient Genius*, 64, Springer, 2012.

79. W. Heisenberg, Ueber den anschaulichen Inhalt der quantentheoretischen Kinematik and Mechanik, *Zeitschrift für Physik*, 43: 172–198 (1927). English translation in Wheeler and Zurek 1983: 62–84.

80. W. Schommers, Space-Time and Quantum Phenomena, in *Quantum Theory and Pictures of Reality*, W. Schommers (ed.), Springer, Berlin, 217-277, 1989.

81. M. Jammer, *The Concept of Mass in Classical and Modem Physics*, Harvard University Press, Cambridge, Massachusetts, 1961.

82. H. Poincaré, *Science and Hypothesis*, Dover, New York (1952).

83. A. Einstein, Does the Inertia of a Body Depend Upon its Energy Content, *The Principle of Relativity*. Dover Publications, New York, (1956).

84. A. Einstein, *The Meaning of Relativity*, Dover Publications, New York, (1956).

85. H. Helmholtz, (Leipzig), Uber der Ehaltung der Kraft, Eine Physikalische Abhandlung 18(13), pp. 639–641, 1847.

86. A. Sommerfeld, *Thermodynamics and Statistical Mechanics*, Academic Press, New York, 1956.

87. O. De Pretto, Ipotesi dell'Etere nella Vita dell'Universo, *Reale, Inst. Veneto di Scienze*, *Lettere en Arti* 63(2) 439-500 (1904).

88. L. B. Okun, The concept of mass, Physics Today, April, 31-36 (1989).

89. A. Lorentz, Electromagnetic phenomena in a system moving with any velocity less than that of light, Proc. Acad. Sci. Amst. 6, 809-831 (1904)

90. S. H. Sohrab, Universality of Boltzmann Statistical Mechanics, Thermodynamics, Quantum Mechanics, and Shannon Information Theory, C.H. Skiadas and Y. Dimotikalis (eds.), *14th Chaotic Modeling and Simulation International Conference,* Springer Proceedings in Complexity, 2022.

91. F. Hasenöhrl, Zur Theorie der Strahlung in bewegten Körpern. *Ann. der Phys*. 16, 589-592, 1905.

92. S. H. Sohrab, Invariant laws of thermodynamics and validity of Hasenöhrl mass- energy equivalence formula m = (4/3) E/c^2 at photonic, electrodynamic, and cosmic scales, Bull. Amer. Phys. Soc. 62(1) 124 (2017).

93. J. D. Achenbach, *Wave Propagation in Elastic Solids*, p.124, North-Holland, 1973.

94. P. H. Eberhard, Restoring Locality with Faster-than-light Velocities, in *Waves and Particles in Light and Matter,* Alwyn van der Merwe and Augusto Garuccio (Eds.). Plenum Press, New York,19 (1994).

95. I. Recami, Classical tachyons and possible applications (A review of "Extended Special Relativity", *Nuovo Cim.* 9, No.6, (1986); *Tachyons. Monopoles and Related Topics.* North-Holland, Amsterdam, 1978.

96. V.M. Somsikov, The method of the description of dynamics nonequilibrium systems within the frames of classical mechanics, arxiv: 0710.0078v1 [physics.gen-ph] 29 September, 2007.

97. J. L. Bell, *The Continuous and the Infinitesimal in Mathematics and Philosophy*, Polimetrica, Milano, Italy, 2006.

98. H. Sommerfeld, *Die Bedeutung der Röntgenstrahlen für die heutige Physik*, Munich, 1925, p. 11. Cited in H. Weyl, *Philosophy of Mathematics and Natural Science*, Princeton, 1949.

99. L. Boltzmann, Weitere Studien uber das Warmegleichgewicht unter Gasmoleculen, *Sitzungsberichte Akad.Wiss.,* Vienna, Part II, 66, 1872, pp. 275-370. English translation: G. S. Brush, *Kinetic Theory*, 88-175, Pergamon press, New York, 1965.

100. M. Planck, On the law of the energy distribution in the normal spectrum, Ann. der Phys. 4, 553-558 (1901).

101. B. L. van der Waerden, *Sources of Quantum Mechanics,* B. L. van der Waerden (ed.), Dover, New York, 1967. 95.

102. G. 't Hooft, *The Cellular Automaton Interpretation of Quantum Mechanics*, Springer, 2016.

103. E. Schrödinger, Quantization as a problem of proper values, Part I, Ann. der Phys. 79, (4), 361-376 (1926), Quantization as a problem of proper values, Part II, Ann. der Phys. 79 (4), 489-527 (1926); Part III, Ann. der Phys. 81, 109-139 (1926).

104. W. Schommers, Evolution of Quantum Mechanics, in *Quantum Theory and Pictures of Reality*, W. Schommers (ed.), Springer, Berlin, 1-48, 1989.

105. B. L. van der Waerden, Towards Quantum Mechanics, in: *Sources of Quantum Mechanics,* van der Waerden, B. L. (ed.), Dover: New York, 1967, pp. 1-59.

106. J. T. Cushing, *Quantum Mechanics*, the University of Chicago Press, Chicago, 1994.

107. F. Laloe, and J. H. Freed, The effects of spin in gases. Sci. American 258(4) 94-101 (1988).

108. R. E. Sonntag, and G. J. van Wylen, *Fundamentals of Statistical Thermodynamics; John* Wiley, New York, 1966.

109. M. Kardar, *Statistical Physics of Particles;* Cambridge University Press, New York, 2007.

110. S. H. Sohrab, The physical foundation of a grand unified statistical theory of fields and the invariant Schrödinger equation. *WSEAS* Transactions on Circuits and Systems 3(4), 1017-1025 (2004).

111. M. Dersarkissian, Does wave-particle duality apply to galaxies? Lett. Nuovo Cim. 40(2), 390-394 (1984).

112. M. Born, Zur Quantenmechanik der Stoßvorgänge, *Z. Physik* 37, 863*,* 1926.

113. F. Selleri, Wave-Particle Duality: Recent Proposals for the Detection of Empty Waves, in: *Quantum Theory and Pictures of Reality*, W. Schommers (ed.), Springer, Berlin, 279-332, 1989.

114. J.S. Bell, Six Possible Worlds of Quantum Mechanics, in: *On the Foundations of Quantum Mechanics,* M. Bell, K. Gottfried and M. Veltman (Eds.). World Scientific Publishing,193 (2001).

115. C. Rovelli, On the meaning of time, *Financial Times*, April 20, 2018.

116. A. J. Kox, and H. F. Schatz, "*A Living Work of Art" The Life and Science of Hendrik Antoon Lorentz,* Oxford University Press, 2021.

117. S. H. Sohrab, Scale invariant model of statistical mechanics and quantum nature of space, time, and dimension, Chaotic Modeling and Simulation (CMSIM) 3, 231-245 (2016).

118. W. Pauli, *Theory of Relativity*, p. 204, Dover, 1958.

119. O. Darrigol, The mystery of the Einstein Poincaré connection, Isis 95(4) 614-626 (2004).

120. P. Galison, *Einstein's Clocks, Poincaré's Maps,* W. W. Norton & Company, New York, 2003.

121. A. A. Penzias, and R. W. Wilson, A measurement of excess antenna temperature at 4080 Mc/s. Astrophysics J. 142, 419-421 (1965).

122. P. J. E. Peebles, Lyman A, Page, Jr., and R, B. Partridge, *Finding the Big Bang, Cambridge* University Press, 2009.

123. D. Lambert, *The Atom of the Universe, The Life and Work of George Lemaître,* Copernicus Center Press, Krakow, 2015.

124. S. J. Prokhovnik, The physical interpretation of special relativity, in *Waves and Particles in Light and Matter,* Alwyn van der Merwe and Augusto Garuccio *(Eds.),* Plenum Press, New York, 85 (1994); *Found. Phys.* 9, 883 (1979).

125. T. Hirosige, The ether problem, the mechanistic world view, and the origins of the theory of relativity, *Historical Studies in the Physical Sciences,* 7(3)-82 (1976).

126. H. A. Lorentz, Electromagnetic Phenomena in A System Moving with Any Velocity Less Than the That of Light, in: *The Theory of Electrons*, Dover, New York, 1952.

127. A. Einstein, On the Electrodynamics of Moving Bodies, in: *The Principle of Relativity*. Dover Publications, New York, (1956).

128. S. H. Sohrab, Some thermodynamic considerations on the physical and quantum nature of space and time, International Conference on Efficiency, Cost, Optimization, Simulation, and Environmental Aspects & Energy Process Systems, Enschede, The Netherlands, July 4-7, 2000. WSEAS Transactions on Mathematics 3(4) (2004).

128. I. Recami, Classical tachyons and possible applications (A review of "Extended Special Relativity", *Nuovo Cim.* 9, No.6, (1986); *Tachyons. Monopoles and Related Topics.* North-Holland, Amsterdam, 1978.

130. A. A. Michelson, and E. W. Morley, On the relative motion of the Earth and the luminiferous ether, Am. J. Sci. 34, p.333 (1887).

131. Dirac, P.A. M., The evolution of the physicist's picture of nature, Sci, Amer. 208(5) 45-53 (1963).

132. H. Poincaré, *Mathematics and Science Last Essay*, p.19, General Books, Memphis, Tennessee, 2010.

133. A. M. P. Dirac, *Directions in Physics*, Wiley, New York, 1978.

134. G. 't Hooft, *In Search of The Ultimate Building Blocks*, p. 161, Cambridge University press, 1998.

135. O. Darrigol, *Electrodynamics from Ampère to Einstein,* Oxford University Press, 2021.

136. J. D. Jackson, *Classical Electrodynamics,* Wiley*,* New York, 1975.

137. P. G. Huray. *Maxwell's Equations*, Wiley, 2010.

138. W. P. Higgs, Broken symmetry and the masses of gauge bosons, *Phys. Rev. Lett.*13(16), 508-509 (1964).

139. L. Bass, and E. Schrödinger, Must mass of photon be zero? *Proc. Roy. Soc.* A 232, 1-6 (1955).

140. S. H. Sohrab, Examples of Applications of an Invariant Statistical Theory of Field to Cosmology, Astrophysics, Hydrodynamics, Electrodynamics, and Photonics, C.H. Skiadas and Y. Dimotikalis (eds.), *15th Chaotic Modeling and Simulation international Conference,* Springer Proceedings in Complexity, 2023.

141. D. Van Delft, and P. Kes, The discovery of superconductivity, Physics Today 66, 9, 38 (2010).

142. M. Lachièze-Rey, *Cosmology, A First Course,* Cambridge University Press, 1995.

143. M. V. Berry, *Principles of Cosmology and Gravitation, IOP* Publishing, Ltd., 1989.

144. P. Collier, *A Most Incomprehensible Thing: Notes Towards a Very Gentle Introduction to the Mathematics of Relativity,* Published by Incomprehensible Books, 3rd Edition, 2001

145. P. J. E. Peebles, *Cosmology's Century, An Insider History of our Modern Understanding of Universe,* Princeton University Press, 2020.

146. S. H. Sohrab, Scale-invariant model of Boltzmann statistical mechanics and generalized thermodynamics, ISTE Open Science Ltd. London, (2021).

147. G. 't Hooft, Quantum gravity as a dissipative deterministic system, Class. Quantum. Grav. 16, 3263-3279 (1999).

148. G. 't Hooft, The fundamental nature of space and time, in: *Approaches to Quantum Gravity, Towards a New Understanding of Space and Time and Matter*, D. Oriti Ed., Cambridge Univ. Press 2009, ISBN 978-0-521-86045-1, pp.13-25.

149. D. J. Fixsen, The temperature of the cosmic microwave background, *Astro. Phys. J.* 707, 916-920 (2009).

150. W. C. Saslaw, P. Crane. The scale dependence of galaxy distribution functions, Astro. Phys. J. 180, 315-319, (1991).

151. W. C. Saslaw, *Distribution of Galaxies.*, Cambridge University Press, 2000.

152. O. Lahav, W. C. Saslaw, S. Biasing and distribution functions for different galaxy types in optical and IRAS catalogs, Astro. Phys. J. 396, 430-444, (1992).

153. M. Dersarkissian, Does wave-particle duality apply to galaxies? Lett. Nuovo Cim. 40(2), 390-394 (1984).

154. A. Einstein, The Foundation of the General Theory of Relativity, in: *The Principle of Relativity*. Dover Publications, New York, (1956).

155. M. R. Nugayev, The genesis of general relativity: Interaction between Einstein's, Abraham's and Nordström's research programs, *Kairos Journal of Philosophy & Science* 19, (2017).

156. M. Arminjon, Scalar theory of gravity as a pressure force, Rev. Roum. Sci. Tech.- Méc. Appl. 42(1-2), 27-57 (1997).

157. M. C. Escher, *The Graphic Work of M. C. Escher*, p. 23, Hawthorn Books, New York, 1960.

158. S. H. Sohrab, Implications of a scale invariant model of statistical mechanics to nonstandard analysis and the wave equation, WSEAS Trans. Math. 7, 3, 95-103, 2008.

159. G. H. J. É. Lemaître, A homogeneous universe of constant mass and growing radius accounting for the radial velocity of extragalactic nebulae, Monthly Notices of the Roy. Astr. Soc. 91(5) 483–490 (1931); PNAS 20(1) 12-17 (1934).

160. H. Everett, III, Relative state formulation of quantum mechanics. Rev. Mod. Phys. 29, 454 (1957).

161 A. H Guth, S. H. H. Tye, Phase transitions and magnetic monopole production in the very early universe. Phys. Rev. Lett. 44(10), 631-634 (1980).

162. A. H. Guth, Inflationary universe: A possible solution to the horizon and flatness problems. Phys. Rev. D 23(2), 347-356 (1981).

163. A. Albrecht, J. P. Steinhardt, Cosmology for grand unified theories with radiatively induced symmetry breaking. Phys. Rev. Lett.48(17), 1220-1223 (1982).

164. A. D. E. Linde, A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems. Phys. Lett. B, 108, 6*94* (1982)*;* 389-393 (1988).

165. M. Kamionkowski, Is the universe (statistically) isotropic? Lecture delivered at physics department of Northwestern University, Evanston, Illinois, March, (2010).

166. K. Gödel, An Example of a New Type of Cosmological Solutions of Einstein's Field Equations of Gravitation, *Rev. Mod. Phys.* 21 (3), 447 (1949).

167. R. P. Kerr, Gravitational field of a spinning mass as an example of algebraically special metrics, *Phys. Rev. Let.* 11 (5), 237 (1963).

168. W. de Sitter, On Einstein's theory of gravitation and its astronomical consequences. Monthly Notices of the Royal Astronomical Society 78 (1) 3-28 (1917).

169. S. B. DeWitt, Quantum mechanics and reality, Physics Today 23(9), 30 (1970).

170. B. S. DeWitt, Quantum gravity, Scientific American 249(6)112 (1983).

171. J. A. Wheeler, Superspace and the Nature of Quantum Geometro-dynamics. *Battelle Rencontres*, C. M. DeWitt, and J. A. Wheeler (Eds)., Benjamin, New York 1968.

172. A. Ashtekar, The winding road to quantum gravity, Current Science 89(12), 2064 (2005).

173. C. Rovelli, and L. Smolin, Loop space representation of quantum general relativity. Nucl. Phys. B331, 80 (1990).

174. S. A. Major, and K. L. Setter, Gravitational statistical mechanics: A model, Class. Quantum Grav. 18(23), 5125–5142 (2001).

175. C. Rovelli, and F. Vidotto, *Covariant Loop Quantum Gravity*, Cambridge University Press, 2015.

176. G. 't Hooft, How does God play dice? (Pre-) determinism at the Planck scale, arXiv:hep-th/0104219v1 25, 2001, Netherlands.

177. S. H. Sohrab, Scale-invariant model of Boltzmann statistical mechanics and generalized thermodynamics, ISTE Open Science Ltd. London, (2021).

178. G. Cirier, A probabilistic approach at the Poincaré-Bendixon problem in R^d, Advances in Pure Mathematics 12, 724-741, (2022).
https://www.scirp.org/journal/apm

179. S. Arrhenius, *Worlds in The Making*, Harper & Brothers, New York. 1908.

180. S. H. Sohrab, Invariant forms of conservation equations for reactive fields and hydro-thermo-diffusive theory of laminar flames, J. Energy Resources Technol. 137, 1-10 (2015).

181. F. Verhulst, Henry Poincaré's Inventions in Dynamical Systems and Topology, in: *The Foundations of Chaos Revisited: From Poincaré to Recent Advancements*, edited by C. H. Skiadas, 1-25, Springer, Switzerland, 2016.