IRSTI: 41.23.3

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COMMENTS ON CRITIQUES OF PLANETARY HYPOTHESES

Abstract. Wolf's supposition that solar activity has a causal relationship with the orbital motion of the planets, further referred to as the planetary hypothesis, from time to time becomes the subject of critiques. In this note we aim to show that the solar spin momentum changes due to the spin-orbit coupling and the critiques devoted to planetary hypothesis are not strictly substantiated to deny it. We have created the program-package *SolAct*, which solves a system of equations of the spin-orbit interaction and allows to follow the sun's angular momentum change over time. Program-package *TiTor* has also been created to calculate the tidal torque exerted on the Sun taking into account its rotation. Using *SolAct* and *TiTor* it is proved that the critiques that claim there are no torques exerted on the Sun to change its spin are incorrect. The calculations show that the theoretical model is capable to simulate the observable variations of the solar rotation, which gives hope that the modeled spin-orbit coupling is plausible. Spin-orbit coupling as a transformer of mechanical chaos to physical, leads to solar activity, which is considered as the response to the chaos of the self-organizing solar system through which it releases itself from the additional energy caused by chaos. If the feedback will cause the solar spin and orbital planes to coincide, the solar activity will be weakened, which can be considered as the physicomechanical evolutionary path of the solar system.

Keywords: Sun, solar activity, planetary hypotheses, spin-orbit coupling, data analysis, periodicity, true and spurious periods.

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ПЛАНЕТАРЛЫҚ ГИПОТЕЗАЛАР ТУРАЛЫ СЫНДАРҒА ТҮСІНІКТЕМЕЛЕР

Аннотация. Вольфтың күн белсенділігі планеталардың орбиталық қозғалысымен себеп-салдарлық байланысы бар деген гипотезасы, бұдан әрі планетарлық гипотеза деп аталады, анда-санда сынға ұшырайды. Бұл жазбада біз күннің өзіндік айналу моменті спин-орбиталық адгезияға байланысты өзгеретінін және планетарлық гипотезаны сынау гипотезаны жоққа шығаруға қатаң негізделмегенін көрсетуге тырысамыз. Біз спин-орбиталық өзара әрекеттесу теңдеулер жүйесін шешетін және уақыт өте келе күннің бұрыштық импульсінің өзгеруін бақылауға мүмкіндік беретін solact бағдарламалар пакетін жасадық. Сондай-ақ, күннің айналуын ескере отырып, тыныс алу моментін есептеу мақсатында Titor бағдарламалар пакеті құрылды. Solact және TiTor-ді қолдана отырып, оның айналуын өзгерту үшін күн сәулесінде айналу моменттері жоқ деген сындар дұрыс емес екендігі дәлелденді. Есептеулер көрсеткендей, теориялық модель күннің айналуындағы елеулі өзгерістерді модельдеуге қабілетті, бұл модельденген спин-орбиталық ілінісу сенімді деген үміт береді. Механикалық хаосты физикалық хаосқа түрлендіргіш ретінде Спин-орбиталық байланыс күн белсенділігіне әкеледі, бұл өзін-өзі ұйымдастыратын Күн жүйесінің хаосына жауап ретінде қарастырылады, ол арқылы хаос тудырған қосымша энергиядан арылады. Егер кері байланыс күннің меншікті және орбиталық айналу жазықтықтарының сәйкес келуіне себеп болса, күн белсенділігі әлсірейді.

Түйін сөздер: Күн, күн белсенділігі, планетарлық гипотезалар, спин-орбиталық өзара әрекеттесу, деректерді талдау, кезеңділік, шынайы және жалған кезеңдер

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КОММЕНТАРИИ К КРИТИЧЕСКИМ ЗАМЕЧАНИЯМ О ПЛАНЕТАРНЫХ ГИПОТЕЗАХ

Аннотация: Гипотеза Вольфа, что солнечная активность имеет причинно-следственную связь с орбитальным движением планет, далее называемое планетарной гипотезой, время от времени становится предметом критики. В этой заметке мы стремимся показывать, что солнечный собственный момент вращения изменяется из-за спин-орбитального сцепления, и критика планетарной гипотезы не строго обоснована, чтобы отрицать гипотезу. Мы создали пакет программ *SolAct*, который решает систему уравнений спин-орбитального взаимодействия и позволяет следить за изменением углового момента солнца с течением времени. Также создан пакет программ *TiTor*, с целью вычислить приливный вращающий момент с учетом вращения Солнца. Используя *SolAct* и *TiTor* доказано, что критические замечания, которые утверждают, что нет никаких вращающих моментов, приложенных на Солнце, чтобы изменить его вращение, являются неправильными. Вычисления показывают, что теоретическая модель способна моделировать заметные изменения солнечного вращения, которое дает надежду, что смоделированное спин-орбитальное сцепление является правдоподобным. Спин-орбитальное сцепление как преобразователь механического хаоса к физическому, приводит к солнечной активности, что рассматривается как ответ на хаос самоорганизующейся солнечной системы, через который она избавляется от дополнительной энергии, вызванной хаосом. Если обратная связь заставит совпадать плоскостей солнечного собственного и орбитального вращений, солнечная активность будет ослаблена.

Ключевые слова: Солнце, солнечная активность, планетарные гипотезы, спин-орбитальное взаимодействие, анализ данных, периодичность, истинные и ложные периоды.

on the surface, and can be more easily met with; truth is hid
in great depths, the way to seek does not appear to all the world.
Johann Wolfgang von Goethe

Introduction

Planetary influence on solar activity is for a long time standing challenge Wolf [68], Brown [22], Schuster [60], José [41], Wood and Wood [69], Wood [70]; Blizard [17-19], Fairbridge and Shirley [33], Sperber and Fairbridge [63], Gokhale and Javaraiah [35], Zaqarashvili [73], Charvatova [27], Juckett [42-43]. Planetary hypotheses have passed three periods of development, from the supposition that the solar activity has a causal relationship with the orbital motion of the planets (Wolf [68]), more, solar activity correlates with the movement of the Sun around the center of mass of the solar system (José [41]), to the idea that there is an interaction between the solar spin momentum and the solar orbital angular momentum Blizard [17], Zaqarashvili [73], Juckett [42-43], Aliyev [3- 6].

Remark 1**:** *A lot of papers have been devoted to the planetary hypothesis, but our purpose in this note is not to give a self-contained review over these studies. Herein we confine ourselves to what has been said above.*

As a continuation of these ideas, in 2012 Abreu et al. [1] have published a paper entitled “Is there a planetary influence on solar activity?”, in which the authors have put forward the idea that long term fluctuations of solar activity are probably connected with oscillations caused by the planetary tidal torque exerted on the solar tachocline. They actually, by introducing a new torque, develop the idea of Juckett [42], which is expressed as follows: “The dominance of the low frequency $P(t)$ components in the production of sunspot - relevant cycles from the modulation of $L(t)$ suggests that the sunspot producing mechanism (dynamo?) resonates at these product frequencies" (here $L(t)$ is the solar orbital angular momentum, and $P(t)$ the normalized projection of the solar spin axis onto its orbital radius vector; seemingly, Abreu et al. did not know about this paper of Juckett).

Less than a year later, Charbonneau [26] has favored Abreu et al. [1] with the hope that if the suggested model is true, then a way to solve the open question will be found: "Should it be vindicated, a solid basis for long-term forecasting (and backcasting) of solar activity could then exist".

One more year later Poluianov and Usoskin [55] gave a critical comment on the paper of Abreu et al. [1]. Prior to that, there were other critical remarks by De Jager and Versteegh [28] and Shirley [61] related to the planetary hypothesis. If the critical remarks of De Jager and Versteegh and Shirley concern the mechanics and "physics" of planetary hypotheses, the paper by Poluianov and Usoskin criticizes Abreu et al. due to the spectral analysis performed by them. Even if to prove that the periods of the proxy data and the planetary tidal torque do not coincide, it cannot deny at all the influence of the planets on solar activity.

We find critiques of De Jager and Versteegh [28] and Shirley [61] as unfounded, of course, for different reasons. The same can be said about the paper of Poluianov and Usoskin [56] too.

Notice that opponents of Abreu et al. was concentrated in criticism of their statistical analysis and thus have distracted attention from the new idea of tidal torque (Quite recently, we knew that Juckett (private communication) in his unpublished paper (which has been withdrawn from review, without prejudice, from the journal of Solar Physic in 2005) put forward the idea that planetary induced torques modulate an intrinsic solar oscillation), which is likely will have applications. The last can be included into the third period of development of ideas about the spin-orbit interaction, of course, after generalization to the case when the solar rotation is taken into account.

Before discussing various papers, let's briefly formulate first the basic principles of motion in a rotating system and, in particular, in the solar system.

Materials And Methods. Mechanics Of The Rotating System And The Solar System Mechanics

Denote by B_t the rotation of the coordinate system K_m with respect to the reference coordinate system K_r at rest. Let R and r are the radius vectors of the point in the moving and reference coordinate systems, respectively, $r(t) = B_t R(t)$, and the angular velocity vector in the moving coordinate system. Let $[\cdot,\cdot]$ denotes the vector product, and overdot stand for derivative with respect to t, $\dot{f} = df/dt$. Then the following theorems are true, Arnold [14, pages 130, 143] (see also Landau and Lifshitz [44] and Banach [15]):

Theorem 1: *Motion in a rotating coordinate system takes place as if three additional inertial forces acted on every moving point R of mass m*:

1) *the inertial force of rotation*: $m[\dot{\Omega}, R]$;
2) *the Coriolis force*: $2m[\Omega, \dot{R}]$, and
3) *the centrifugal* force: $m[\Omega, [\Omega, R]]$.

Thus,

$$m\ddot{R} = F - m[\dot{\Omega}, R] - 2m[\Omega, \dot{R}] - m[\Omega, [\Omega, R]] ,$$

where $BF(R, \dot{R}) = F(r, \dot{r}),\ F(r, \dot{r}) = m\ddot{r}$.

Theorem 2: *Let L be the angular momentum in the moving coordinate system and M the sum of moments of the external forces acting on the body. Then*

$$dL/dt = [L, \Omega] + M. \quad (1)$$

Equation (1) with M added is usually called the modified Euler equation.

These two theorems enable us to correctly describe the motion in non-inertial rotating system, in particular, in solar system.

Motion in a rotating system and spin-orbit interaction

Let's first shortly concern the mechanics of the motion of the Sun. During the motion around the barycenter, the trajectory of the solar center is so complex that even torsion changes not only the value, but also the sign. Moreover, the radius of the sun's trajectory around the barycenter is comparable with the radius of the Sun, which means that the trajectory of Sun's center is strongly curved. To think that a gyroscope as the sun during such a walking will not be subjected to any influence and will preserve its state, is not reasonable. The picture is as follows: The sun is immersed into a non-inertial system and, therefore, will be under the influence of additional forces (see Figure 1 and Figure 2). The sun is a gyroscope that rotates, oscillates and spirals with variable torsion.

One of the forces must be arisen because of that the solar gyroscope is forced to simultaneously rotate around the center of the Sun and the center of mass. If we recall that the center of mass of the solar system is inside the sun, it is quite clear that a force with a moment arm equal to the distance between the solar center and the center of mass will certainly generate a torque that will change the solar spin.

There are other torques (for example, $[L,\Omega]$, where L is the spin momentum of Sun and the instantaneous orbital angular velocity) to change the solar rotational momentum, but the previous torque plays a major role in the spin-orbit interaction.

This vision opens the window to look at old astrophysical problems again, for example, such as why close binary systems, in general, multibody systems are so active.

In the notes Aliyev [7, 9-10], see also Appendix A) we have reported on the program - package *SolAct* (Module Solar Activity), which was created to generate the motion of the Sun and the planets around the solar system barycenter, to calculate the angular momenta and orbital parameters. Moreover, *SolAct* solves the system of equations of spin-orbit interaction and allows us to follow the sun's angular momentum change over time.

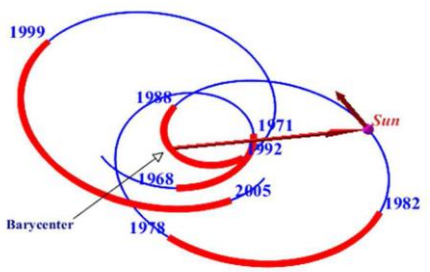


Figure 1 – Solar center motion in 1966 – 2006 (3D plot, precisely 3D animation). Trajectory parts in red show for years of activity.

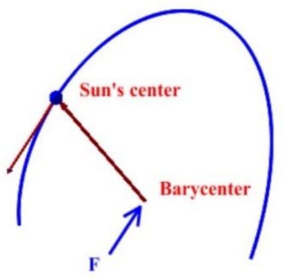


Figure 2 – The moment arm between the barycenter and the solar center and the force applied to the sun at the point of the mass center.

We call the rotating system of the Sun and Jupiter, with embedded all other planets, as the solar system catastrophe machine because of the similarity to the Zeeman catastrophe machine.

However, the solar system catastrophe machine is very complex than that. The main problem is how to determine the torque, moreover how to compute it. For this purpose, a new version of the module *SolAct* was created. *SolAct* takes into account the torques exerted on the interior of the sun (more precisely, on rigidly rotating part of the sun) from the outer planets.

Assume that the sun is under the influence of the torques of outer planets. A torque of the type $[L,\Omega]$ is also added. The calculation that uses a time-dependent coefficient for the spin-orbit coupling, shows, as it is illustrated in Figure 3, that the solar angular velocity and Wolf numbers are in remarkable correlation (anti-correlation):

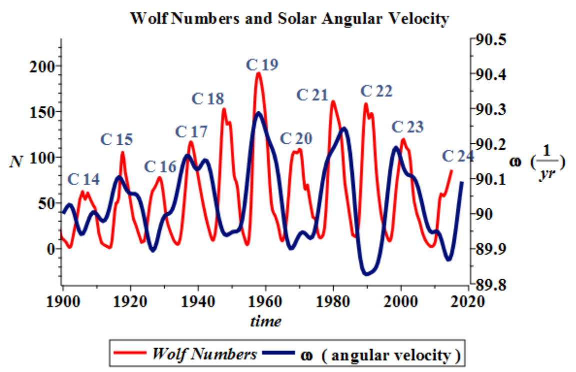


Figure 3 - Solar angular velocity, ω (blue), and Wolf numbers, N (red): the case of variable coupling coefficients.

The theory requires proxy data for calibration. However, the absence of consistent experimental results is an obstacle to the correct construction of theoretical models. Note that results of papers by Javaraiah [38], Javaraiah et al. [39-40], Brajša et al. [20], Jurdana-Šepić et al. [44], Xie et al. [71], Li et al. [46-47], Zhang et al. [74-75], which relate to the solar rotation rate, contradict each other, in one case having a difference in kind, in another case in numerical results. With respect to such inconsistencies Suzuki [61] notes that "*So we do not yet have an observationally conclusive result on the relation between sunspot activity and the differential rotation*".

Antia and Basu [11], and Antia et al. [12] by inversion of GONG and MDI data (covering the time interval of 1995-1999 in the first paper and 1995-2007 in the second one) found out that the solar rotation rate shows a clear change with solar activity and they state a remarkable result that there are zonal flow bands of faster and slower rotation, which move to the equatorward with time as the solar cycle progresses. Antia et al. [13] studied the solar interior rotational rate and showed a significant temporal variation in the angular momentum and rotational kinetic energy.

But noisy data from the tachocline region ($0.70\,R \le r \le 0.74\,R$, where R is the solar radius) does not allow to infer consistent information for the angular momentum variation, since the error bars have the order of the data variation.

For the first approximation, we used results of Javaraiah [38] which establish that the decrease in the solar equatorial rotation rate from cycle 17 to cycle 18 is about $0.008\ \mu rad \cdot s^{-1}$, and from cycle 21 to cycle 22 about $0.016\mu rad \cdot s^{-1}$. The data analysis carried out by Javaraiah [38] and Javaraiah et al. [39], (see also Wilson et al. [66]), showed that the rate of solar equatorial rotation is relatively lower in even cycles than in odd cycles. The same result can be inferred from the work by Lustig [49, Figure 5] for a period from 1947 to 1981. However, the opposite result that the equatorial velocities increase, has been established in that paper. These data are given for the solar surface and therefore we have confined ourselves to use what is available.

In the above theoretical consideration, the centennial trend of the solar rotation rate first raises from cycle 14 to 19, then falls to cycle 24. Wolf numbers show the same temporal trend. It is interesting that this corresponds to the result of Li et al. [46] (there is a contradiction between the results of Li et al. [46] and Javaraiah et al. [40]. The last paper shows that the trend of the solar rotation rate goes down from 1879 to 1975. Li et al. [46] suggest that the contradiction is probably connected with the various methods used for the data analysis. Later on Li et al. [47] refused the results). Figure 4 shows the time dependence of secular trends of the solar rotation period and Wolf numbers. As seen from Figure 4, the minimum of the trend of the solar rotation period exactly coincides with that obtained by Li et al. [46], i.e., with the cycle 19.

Such theoretical model is one of the possible realizations. To gain the secular deceleration of the solar rotation rate till now, as it is declared, for example, by Javaraiah et al. [40] and Brajša et al. [21], it is necessary to change the initial values and coupling coefficients. Then the model gives the values shown in figure 5.

In both above cases (see Figures 3 and 5), odd cycles correspond to the maxima of solar angular velocity, while even cycles to the minima. The model is sensitive to both the initial values and coupling coefficients, and allows alternative possibilities. There would be consistent observational results for calibration.

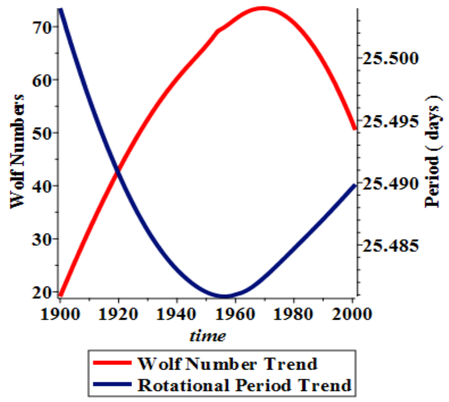


Figure 4 - Trends of Wolf numbers and the solar rotation period.

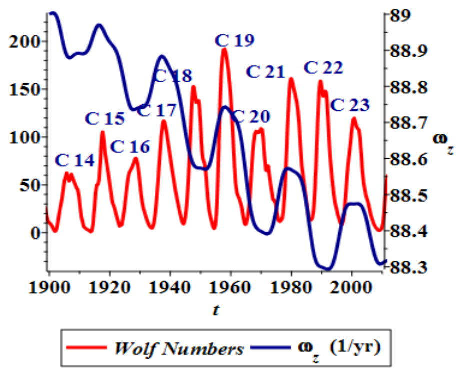


Figure 5 - Solar angular velocity, ω_z (blue), and Wolf numbers, N (red): the case of secular slowing of the solar rotation rate.

The velocity field in the solar envelope

Here we have as yet considered only the mechanics of the Sun and, the theory which establishes a connection between long-term variations of the solar spin and solar activity still waits its development. The construction of a dynamo model requires to know the velocity field formed in the solar envelope under the influence of the solar irregularly rotating and wobbling rigid core.

Full solution of the problem is difficult enough. Therefore, here we will confine ourselves to consider the area adjoining to the solar rigidly rotating core, other words, the bottom of the tachocline. Suppose that the tachocline is in balance, the fluid is incompressible, the macroscopic velocity field considered in that area is $V = [\Omega(t), r]$, solar oblateness is not taken into account, $R_x^{\odot} = R_y^{\odot} = R_z^{\odot}$, where $R^{\odot}$ is the solar radius in the bottom of the tachocline, $|r| \geq R^{\odot}$. Consider the perturbation of the velocity field and neglect the perturbations of gravity and pressure. Neglect the x and y components of the solar rigid core angular velocity ($\Omega_x = 0$, $\Omega_y = 0$) and z −dependence of the functions, then the velocity field in the cylindrical coordinates enclosing the solar center, reads

$$\upsilon_x\,(r,\varphi,\tau) = F_1\,(r,\varphi+\tau)\cos\,(2\,\varphi),$$
$$\upsilon_y\,(r,\varphi,\tau) = F_2\,(r,\varphi+\tau)\,\sin\,(2\,\varphi),$$
$$\upsilon_z\,(r,\varphi,\tau) = F_3\,(r,\varphi+\tau)\,. \quad (2)$$

Here $\tau = \int \Omega_z\,(t)dt$ and $\Omega_z(t)$ is z −component of the angular velocity, $\boldsymbol{r} = (r,\varphi,z_0)$ and φ – is the azimuthal angle. F_i with $i = 1..3$, are the arbitrary functions which can be determined from the given initial and boundary conditions. In such form of Eq. (2), the velocity field represents traveling waves spiraling around the z − direction. Such solutions show on the existence of torsional waves having an 11-year and/or 22-year periodicity aside from short-term oscillation.

In the above simplified case, the equation of motion admits also solutions that depend only on time. Using this fact we have numerically solved the equations taking into account that $\Omega_x(t) \neq 0$, $\Omega_y(t) \neq 0$ and for various initial conditions to explicitly show the time dependence of the velocities (Figure 6).

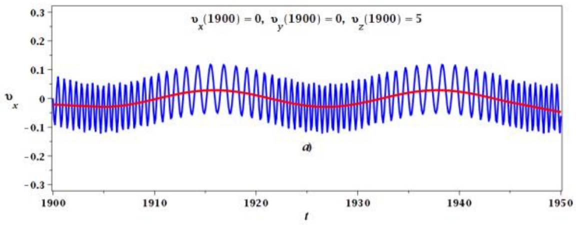
Note some important properties that velocities possess. Solutions are sensitive to initial values and coupling coefficients. Therefore, the following properties are constrained by the accepted conditions. Figure 6 shows that,

(a) the velocities υ_x and υ_y show a 22-year periodicity, while the period of υ_z is 11-year and their amplitudes of long-term variations increase when the solar cycle goes to a maximum;

(b) oscillations of υ_z cyclically either go down, or become stronger. The time course of the average υ_z during one cycle very similar to that has been found by Hathaway et.al. [36, Figure 4], (see also Basu and Antia [16] and references therein);

(c) the frequency of short-term oscillations undergoes a cyclical variation;

(d) the υ_x and υ_y simultaneously change sign with the period 11 year. Consequently, the travelling waves will cyclically change the direction of propagation corresponding to the behavior of torsional oscillations. Probably, it will also concern cyclonic perturbations, which will cyclically change the direction of vorticity in both hemispheres.



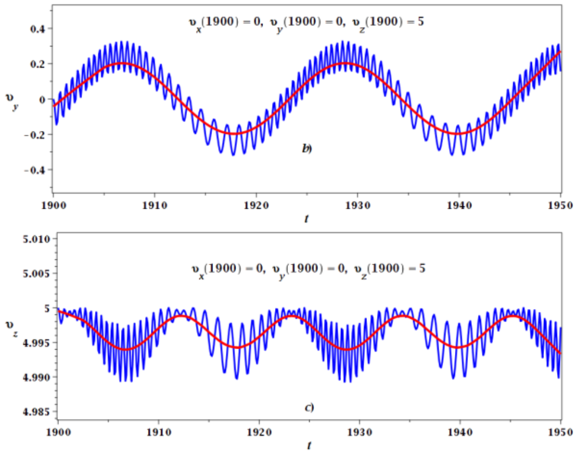


Figure 6 - Time dependence of dimensionless velocities, v_x, v_y and v_z with arbitrary normalization (blue), and their average with respect to the short-term oscillation (red).

The main conclusion that can be obtained from the above properties is that the perturbed velocity field is amplified during the activity maxima and attenuated during the minima, which hints at why the local magnetic fields are cyclically arose, for whose generation the perturbed velocity field is responsible.

Note that the curves in Figure 6 are regular, only because we have fitted $\Omega_x(t), \Omega_y(t)$, $\Omega_z(t)$ to periodic functions, removing the trend and not trying to reach exact coincidence of amplitudes in order to facilitate calculations.

Opportunities

When masses are comparable, a powerful mechanical chaos in the many-body system is inevitable. The solar system many-body problem is solvable because $m_i/M_{\odot} \ll 1$ and any F_{ij} (forces between two pairs of planets) is negligible. Mechanical chaos also occurs in the solar system. This disorder there is due to that the condition $m_i/M_{\odot} \ll 1$, where $i \geq 5$, is not satisfied for the outer planets, which leads to that the Sun's orbit about the barycenter is not closed, but the degree of such chaos is small. Just that deterministic chaos through the spin-orbit coupling forces the rotating rigid Sun and its envelope to wobble. In our opinion, solar activity is a response to the chaos of the self-organizing solar system, through which it releases itself from the additional energy caused by chaos. If the feedback will cause the solar spin and orbital planes to coincide (see below), the solar activity will be weakened. It, we believe, is the natural physicomechanical evolutionary path of the solar system.

Why to model the solar activity on the basis of theoretical inventions, rather than to use experimental data? Observations show on the tilt of solar dipole with respect to the rotation axis (Wang [66], Norton et al. [52], Yabar et al. [72]). Solar activity occurs in most cases at syzygy of Jupiter and Saturn with respect to the Sun. There are torsional waves that cyclically change directions. The two hemispheres of the solar surface show north-south asymmetry. Odd and even cycles differ from each other. The reversal of the magnetic poles occurs at the maxima of solar activity, and almost at the same time the predominance in north-south asymmetry changes.

Even before to completely solve the problem, the suggested mechanical model of spin-orbit coupling makes it possible to have some notions concerning the observed phenomena of solar activity, for example, why does solar activity occur at syzygy of Jupiter and Saturn? (Solutions of equation, $dL/dt = 0$, define points of bifurcations. Jupiter and Saturn bring the biggest contribution at the moment of forces and when they are on one line, the moments of forces appear close to zero. In points of bifurcations the mechanical chaos in system begins and it is transformed to physical chaos which is observed as solar activity.), why solar activity is cyclical and why the cycle length is on average 11 years (solar activity is the result of deterministic chaos caused by cyclic planetary influence on the Sun with an average cycle duration of 11 years), why there is a north-south asymmetry (the gyroscope inside the Sun changes the rotation plane during the 11-year cycle, at the same time oblate tachocline, which is responsible for solar activity, changes the plane of rotation, which leads to spatial asymmetry with respect to the solar equator), why do even and odd cycles differ from each other (odd cycles correspond to maxima (minima) of the solar spin momentum, but the even ones to minima (maxima)), why do torsional oscillations arise and change the direction of propagation from east to west and vice versa with periods of 11 years (the solar core twists around the mass center cyclically and forces the velocity field in the solar envelope to torsionally oscillate).

To throw light on puzzling question, are there discontinuities in the solar activity for long time intervals, let's act as follows. The cosine of the angle between the solar spin momentum, L, and the angular velocity, Ω, (the cosine of the angle in the future will simply be called the angle) has the same evolution course as the angular velocity (see Figure 3), but the former is more informative.

During precession the solar activity is strong if the angle has a change of relatively large values, and is weak if the change is of small amplitude, for example as in cycles 14 and 15.

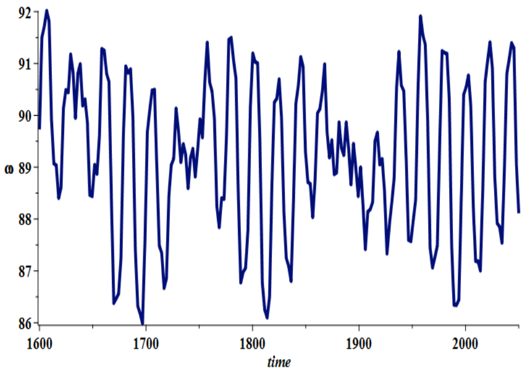


Figure 7 - Solar angular velocity for a time interval of 1600-2050.

It is well seen from Figure 7 that there are epochs of strong and weak oscillations of the solar angular velocity (note that the extrema of Wolf numbers and the angular velocity can be displaced outside the interval 1900-2010, since the coupling coefficients are calculated only for this interval). It means when the Sun orbits the barycenter and the angle oscillates of insignificant amplitude, but the torsion changes sign, then the Sun will show weak activity. In this case the precession of the solar spin momentum is almost stationary, in other words, there is almost no twisting of the orbit, or it is possible that L is commonly parallel to Ω (if to neglect the free precession due to the solar oblateness), i.e., the total torque is zero.

One important conclusion from the above analysis is that precession along with the torsion makes the Sun's orientation asymmetrical with respect to the rotational axis. In this formulation, the Sun can be considered as co-rotating system of variably rotating and precessing core and envelope, and such approach can be regarded as a posing of the problem. In other words, there exists a spin-orbit coupling which leads to a variation of the solar rotational rate. The wobbling solar rigid core forces the solar envelope to wobble also. It leads to a cyclically variable velocity field in the solar envelope, which is responsible for ensuring the operation of the solar dynamo. We abandon the kinematic dynamo and intend to use a magnetohydrodynamical dynamo, which unlike the kinematic dynamo will be self-consistent due to that the velocity field is found from the momentum equation.

It is very much like the phenomena in behaviour of the fluid contained in rotating and precessing cylinder described by Mouhali et al. [51]: ... *when* ε *is increased from small values, we have observed an induced differential rotation followed by the apparition of permanent cyclonic vortices*. Here $\varepsilon = \Omega_p / \Omega_0 \ (\ll 1)$, where Ω_0 is the angular velocity around the z direction, and Ω_p the angular velocity of precession.

True ideas do not disappear, they either rehabilitate oneself, or are born again. Wolf's planetary hypothesis after nearly 100 years, in the 40s of the 20th century, was born again in the theory of terrestrial magnetism. To explain the Earth's magnetism, Elsasser [30-32] put forward the dynamo theory together with a "planetary hypothesis", that the lunar tide could be a source for maintaining the Earth's magnetic field. It was wonderful that the dynamo theory appeared together with the planetary hypothesis. Elsasser's planetary hypothesis in the theory of terrestrial magnetism has been developed now up to the idea that earth precession should be taken into account and the question has been put so: Is precession the cause of geomagnetism? (Malkus [50], Rochester et al. [58]). This way of investigation is still going on (Roberts and Stewartson [56], Glatzmaier and Roberts [34], Channell et al. [23], Stefani et al. [64], Dormy & Le Mouël [29], Lin et al. [48]). The center of mass of the Earth-Moon system is inside the Earth and the torque associated with it will be incomparably greater than the lunar tidal torque. This fact opens a new horizon for the theory of terrestrial magnetism.

But, the solar dynamo theory has gone on other way of development though it had its rise from the idea of Elsasser (see Parker [53-54). Isn't it time that two ideas of a dynamo and a planetary influence have been united in the theory of solar activity?

Here we have more dwelt on the mechanics of solar system and the above calculations show that the model of spin-orbit coupling is capable to simulate the observable variations of the solar rotation, which raises one's hope that the theoretical model is believable. With the lapse of time, the idea that the solar rotation is governed by the planetary torque, will not raise doubts. The point here is how much can we correctly model this complex challenge. We think that it is only the commencement, and the theory will pass many tests. Theoretical calculations require to know the coupling coefficients and how to correctly give the initial value of the angular velocity for some time. In this way, the helioseismic analysis of the variation of solar deep rotation would be the best helper for calibration. Though results of a helioseismology of deep layers show on the prolate core of rotating fluid which is a "little doubtful".

Comments On The Criticisms Of De Jager And Versteegh, And Shirley

In the paper by De Jager and Versteegh [28], the authors "examine ... hypothesis" whether "solar activity originates by planetary Newtonian attraction on the Sun" and come to the conclusion "that the cause of the dynamo is purely solar". The authors used the historical prerequisite that the tidal force define the nature of solar activity and compared it with the corresponding force of the dynamo model. No dynamics is considered by them and the problem is too oversimplified to become the subject of extensive discussion. Their arguments to reject the planetary hypothesis become ineffective due to the discussions in the previous section. Can such a challenge as solar activity be explored by means of simple concepts (also by primitive calculations) at present?

Contrasting of two incomparable concepts is typical of most critical remarks devoted to the planetary hypothesis. How can these two concepts - the planetary and the dynamo - be contrasted while one of them is under the hypothesis, and the other allows within the permissible assumptions to replicate the solar cycles? Who can argue that these two concepts cannot survive together and complement each other? Are the existing dynamo theories of the solar cycle so perfect and really they do not experience certain problems (e.g., see Charbonneau [25], Spruit [62], note that the list of critical remarks can be supplemented)?

Shirley [61] criticizes Zaqarashvili [73] for the reason that he includes into the system of dynamo equations the force that arises inside the Sun as the Sun orbits the Sun-Jupiter center of mass, and Juckett [42] for the reason he puts forward the idea about the existence of a spin-orbit interaction. Shirley [61] writes that "the Sun's orbital motion is a state of free fall ...", and consequently, "... there can be no relative acceleration" ... "due to the revolution of the Sun about the Solar system barycenter; and the spin-orbit coupling hypothesis of Zaqarashvili [73] must be discarded."

Such a statement about the idea of Zaqarashvili [73] is not strictly substantiated and rejected due to Theorem 1. The same can be also said with respect to the "disqualification" (Shirley [61]) of the spin-orbit interaction mechanism Juckett [42], and is easily rejected due to Theorem 1 and Theorem 2, and the discussion in Section 2. However, each of these cases requires detailed discussion.

In the case of Zaqarashvili all is clear: there exists forces (see Theorem 1), but it is necessary to find out how much these forces are hard linked to the system. Note that a force of the type $\rho[[\omega,\Omega\,],r]$ ($=\rho[\dot{\omega},r]$) which is caused inside the fluid rotating with angular velocity ω by the external torque, has been considered as far back as by Poincaré [55].

Concerning the idea of Juckett, Shirley [61] notes that "to alter the rotation state", it is necessary "a force with a non-vanishing moment arm".

So it is indeed. If the center of mass is inside the Sun, then the solar gyroscope is forced to also rotate around the other axis. It means that at the center of mass, an additional force having an arm equal to the distance from the Sun's center to the center of mass acts on the Sun (see Section 2). Note that the centers of mass of all planets, except Jupiter, are inside the

Sun. Moreover, there exists another torque that occurs if the spin and orbital angular momenta are not parallel, $[L, \Omega]$, where L is the spin momentum and Ω the instantaneous orbital angular velocity of the Sun.

For example, the center of mass of the relative motion of the Sun, Jupiter and Saturn is inside the Sun, and the torque components at this point, for *t* = 2000, have the values:

$$K_x/M_{\odot} = 6.98 \cdot 10^3 \; cm^2/s^2;$$
$$K_y/M_{\odot} = -1.37 \cdot 10^4 \; cm^2/s^2;$$
$$K_z/M_{\odot} = -1.24 \cdot 10^5 \; cm^2/s^2, \quad (3)$$

where $M_{\odot}$ is the solar mass.

Again the point is that how much the torques are hard coupled to the Sun. This is similar to the case when the Foucault pendulum is not hard connected to the dome, and therefore the Coriolis force must be introduced into the equations with some coefficient. Some cases of such consideration are implemented using the module *SolAct*, which is briefly described in Section 2 (see also Aliyev [7, 9-10] and Appendix A).

Comment On The Abreu Et Al. Paper

Despite the fact that the work of Abreu et al. [1] is not devoted to criticism of the planetary hypothesis, first of all, on the contrary, we decided to analyze it in detail to reveal its advantages, so that the opponents do not limit themselves to criticize the paper, but they level criticism at the planetary hypothesis as well.

It is necessary to emphasize the remarkable idea of tidal torque advanced by Abreu et al. (2012), but at the same time, it should be stressed that the model proposed by them is not worked out quite:

(a) spatial orientation of the tachocline ellipsoid is not linked to the realistic position of the solar axis;

(b) the solar rotation is not taken into account.

1. In the paper of Abreu et al., solar rotation is ignored, and as a result, the rotation of the tachocline ellipsoidal shell is not considered. The program-package *TiTor* (Tidal Torque, see Appendix B) will help us take the solar rotation in the calculations of the tidal torque into consideration. Note that, when we will compare two cases with and without solar rotation, the spatial position of the solar rotation axis will be ignored.

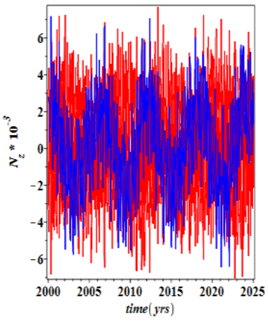
It is obvious that in the case (when the solar rotation is ignored) of a prolate ($R_x = R_y, R_z$, $R_x < R_z$) or oblate ($R_z < R_x$) Sun, the N_z component of the tidal torque will be zero (this is the same with that designated (3) by Abreu et al.). Therefore, for the theoretical experiment when the solar rotation is disregarded, we consider the case of $R_x \neq R_y = R_z, R_x > R_z$, as in the case (1) by Abreu et al. In Figure 8(a) the full tidal torque[1] (all planets, z −component) with the Sun's rotation (red) and without it (blue) are compared. Evidently, the solar rotation strongly modulates the tidal torque. It is interesting, as seen from Figure 8(b), that the tidal torque with solar rotation averaged over the solar rotation period (red) is almost negligible. The physics of such result is that for one quarter of the solar rotation period tidal torque will accelerate tachocline, and during the other Moreover, short–term modulation will transform slow long–term oscillations into noise (see Figure 8(a), the red curve), and such a slight tidal torque with nearly zero short–term mean value (see Figure 8(b), the red curve), along with other randomly added noises in the tachocline region, is unlikely to be able to carry memory.

2. However, the case is different. In fact, Abreu et al. consider the tidal torque without short-term modulation (see Abreu et al. [1]):

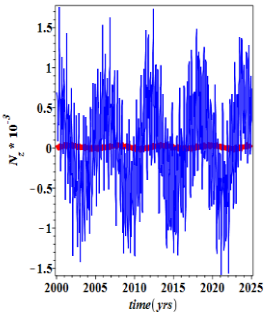
$$N_{z,i} \propto m_i x_i \, y_i/|r_i|^5 \propto m_i/a_i^3 \, f\,(t), \quad (4)$$

where m_i, x_i and y_i denote the mass and heliocentric coordinates of the ith planet, respectively, and a_i and r_i are the semi-major quarter it will slow it down, which is clearly seen, for example, from the course of the Venus tidal torque, which is illustrated in Figure 8(c) axis and the position vector of the ith planet; f is some function of t.

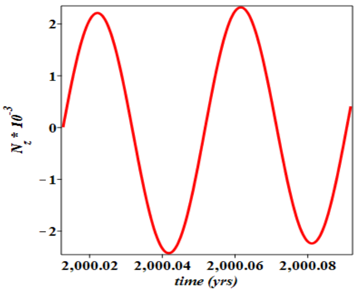
[1] Throughout in this paper, the "full tidal torque" and the "total tidal torque" are used in the same sense, namely, in the sense that all planets are taken into account.



(a)



(b)



(c)

Figure 8 - a) Total tidal torque with solar rotation (red) and without it (blue), b) Total tidal torque with solar rotation averaged over the solar rotation period (red) and the same (without solar rotation) but not averaged (blue), c) Tidal torque with the sun's rotation for Venus at a time interval comparable with solar rotation period. ■ Tidal torques are calculated for $1g$ of solar mass and a unit is $[g\ cm^2/s^2]$.

If to come back to history, such a formula has already been met, although to describe another quantity. Wolf is the first (Wolf [68], Charbonneau [24]) to introduce a formula for the quantitative description of the evolution of sunspots over time. In 1859 he gave a formula for the sunspot number, M,

$$M = 50.31 + 3.73 \sum_1^4 \frac{m_i}{r_i^2} sin(2\pi/T_i \cdot t), \quad (5)$$

where m_i, r_i and $T_i$being planetary masses, mean distances and orbital periods of Venus, Earth, Jupiter, and Saturn, are normalized to the earth's mass, the earth-sun mean distance and the earth's orbital period, respectively.

But, in these cases of formulae (4) and (5), the long-term periodicity is preserved.

3. In the above remark 1 we generalized the formula of Abreu et al. to the case of solar rotation and in order for it to cease to be exotic and enter into the equations of the spin-orbit interaction (Section 2), it is sufficient that the rigidly rotating part of the Sun had an ellipsoidal shape. The problem becomes more realistic thanks to observations that point to the oblate spheroidal shape of the Sun (see, for example, Howe (2009), and references therein), $R_x = R_y, R_z, R_z < R_x$, with $R_z = R_x - 6$ (km). It is a slight difference between the semi-major axes, but it (together with an inclination of the Sun's rotation axis to the ecliptic) makes N_x, N_y and N_z different from zero. Thus, we assume that,

(a) Sun's rotation axis is tilted with respect to the ecliptic for $i = 7.25^{\circ}$ and the longitude of the ascending node of the intersection of the Sun's equatorial plane with the ecliptic is accepted to be $\Omega = 75.76^{\circ}$;

(b) The sun consists of a rigidly rotating internal part and a convective envelope. It is supposed that the tidal torque is applied to the rigidly rotating interior and the convective exterior separately, at that the densities of the internal and external parts are assumed, for the first approximation, to be constant and equal to the mean ρ_r and ρ_c, consequently (rigorous approach needs to know coordinate dependence of the density and the differential rotation of the convection zone);

(c) The shape of the rigidly rotating part of the Sun is an oblate ellipsoid, and its semi-major axes are taken to be $R_x \equiv R_y \approx 0.7 R_{\odot}$ and $R_z = R_x - 100\ (km)$.

The calculation for the case of an oblate sun with rotation (only for the tidal torque exerted on the rigidly rotating part) is shown in Figure 9.

If we compare the tidal torque (see the ordinates of Figure 9) with the torque from Equation (3), we observe that the tidal torque is incomparably small. Changing the value of Z in $R_z = R_x - Z\ (km)$ from 100 to 1000 results in a slight increase of the tidal torque. Now, the tidal torque can be included into the system of spin-orbit coupling equations, however we know that its contribution to the final result will be insignificant.

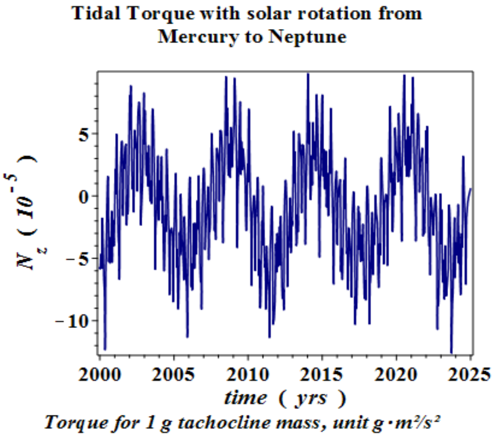


Figure 9 – Total tidal torque exerted on the rotating oblate sun.

Results. Some Problems Of The Data Analysis And Remarks On The Poluianov And Usoskin Paper

Critique regarding spectral analysis carried out by Abreu et al. [1], is outlined in a paper by Poluianov and Usoskin [56]. Note that some rebuttals related to this paper already exist (Abreu et al. [2], Scafetta et al. [59]). Despite it we have decided to dwell on it in detail, since there are some issues concerning the data analysis that go beyond the limits of the examined task and are significant not only for astrophysics.

Abreu et al. [1] show that the cycles of the planetary tidal torque correlate with the long-term cycles in proxies of solar activity. They used annually averaged time series of tidal torque, which became the subject of critical remarks by Poluianov and Usoskin [56].

Poluianov and Usoskin [56] are intended to show that the annual averaging of data does not allow to find long periods correctly and in this regard they note that, "*the spectrum computed from the annually averaged data has nothing in common with the 'true' spectrum*", and continue that "*the spectral peaks in the planetary torque series claimed by A12* (Abreu et al. [1]) *are caused by an artefact of the applied method, viz. the aliasing effect because of the annual averaging of the data before processing*".

Then we have to say that this is not the correct conclusion:

(a) in a multiperiodic case, long periods can be correctly inferred from data averaged over the short-term period;

(b) in a forward problem, all spurious periods can be predicted and separated from true periods;

(c) the data averaging engenders not always spurious periods that can significantly change the spectrum, in addition, it weakens spurious signals;

(d) FFT (Fast Fourier Transform, which is used by Poluianov & Usoskin) adds spurious periods to the spectrum and shifts all periods, so the FFT spectrum cannot be called a 'true spectrum'. The FFT spectrum, primarily, can serve as an example for an artifact generated by the tool used.

Forward problem

Let's first, as an example, perform a spectral analysis of the time series of tidal torque, as it has been presented by Abreu et al. [1]. For this purpose, we will use the program-package *TiTor* (Appendix B). Notice that the periods of planets, which are generated by *TiTor*, may differ slightly from the conventional ones (*TiTor* uses the *JPL* data only once as initial values for any given time, which is the reason for small differences).

Begin with the remark why to search for periods when functions are known, but then the periods are also given! In case of the formula (4) the issue stands exactly so. This is a forward challenge, therefore the search for periods (in the case of data sampling) can serve as a test to validate the method used, and to know how much it reliably carries out the issue and can be robustly applied to data analysis in the future.

Consider the formula (4):

$$\frac{x_i y_i}{|r_i|^5} \Rightarrow \frac{(cos\,(i)-\varepsilon_i)\sqrt{1-\varepsilon_i^2}\,sin\,(\xi_i)}{a_i^3(1-\varepsilon_i cos(\xi_i))^5}.$$

Substituting $\xi_i \approx \omega_i t$ ($\xi_i = \omega_i t + \varepsilon_i \sin(\xi_i)$) into this formula, we obtain (for the full expression see Appendix C),

$$\frac{x_i y_i}{|r_i|^5} \propto -1024\,sin\,(\omega_i t) + (-512\,sin\,(\omega_i t) - 2560\,sin\,(3\omega_i t))\,\varepsilon_i + (-2560\,sin\,(2\omega_i t) - 3840\,sin\,(4\omega_i t))\,\varepsilon_i^2 + (-3840\,sin\,(3\omega_i t) -$$

$$3840\,sin\,(5\omega_i t))\varepsilon_i^3 + (960\,sin\,(2\omega_i t) - 3840\,sin\,(4\omega_i t) - 2880\,sin\,(6\omega_i t))\varepsilon_i^4 + O(sin(\omega_i t), \dots, sin(12\omega_i t), \varepsilon_i, \dots, \varepsilon_i^{11}). \quad (6)$$

Here a_i is the same as it was above designated, ε_i and ω_i are the eccentricity and the circular frequency of the ith planet, respectively. As can be seen from this expression the term with the frequency of $2\omega_i$ ($\omega_i = 2\pi/P_i$), or with the period of $P_i/2$, will have the largest amplitude (because of eccentricities are of small values). Thus, the spectrum will contain the periods of $P_i/n, n \in [1, 2, \dots, 12]$. This means that the greatest period may be only P_8, in other words, the period of Neptune. However, with such a number of periodic functions, other implementations of long periods are possible. The superposition is among them, but it is effective when the amplitudes of the oscillations are comparable.

Remark 2: *TiTor uses the exact time equation. The above simplification in Equation 6 is used for a clear realizing.*

To find out which planet's tidal torque is comparatively large, let's calculate the relative (averaged) amplitudes in terms of Jupiter's tidal torque, N_i/N_J:

Table 1 – Relative amplitudes of planetary tidal torques

Planets	Mercury	Venus	Earth	Mars
N_i/N_J	0.686	0.851	0.851	0.0155
Planets	Jupiter	Saturn	Uranus	Neptune
N_i/N_J	1.0	0.0482	0.00088	0.00025

As seen from Table 1, the Jupiter's tidal torque has the largest amplitude. Then follows the tidal torques of Venus, Mercury, Earth, and Saturn. Now it is clear why Wolf in the formula (5) took into account only the impact of four planets, Venus, Earth, Jupiter, and Saturn (certainly, using the formula m_i/r_i^2 for calculations) and only sinusoids to describe the time dependence.

Rather than to plot the torque modulus $|N(t)|$ for 1000 years, for which the picture looks very vaguely, we consider the time-dependence of $N_x(\equiv 0)$, N_y and N_z for 25 years, where details of temporal evolution are clearly allocated.

Figure 10 shows that the contribution to the total tidal torque from the y –component is insignificant. The reasons for this result are that the orbital planes of the planets are nearly on the ecliptic and the chosen model for the ellipsoid ($R_x \neq R_y = R_z$, $R_x > R_z$).

Moreover, the main periodic variation of a large amplitude relates to the tidal torque of Jupiter, which has an obvious period of $P_5/2$, and all the other curves sit on this curve.

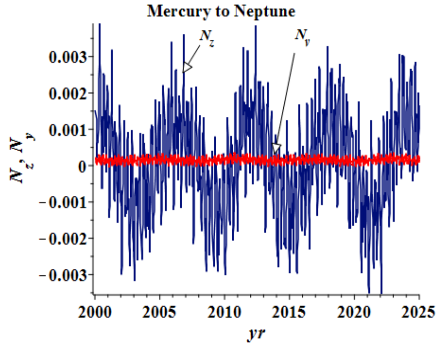


Figure 10 – N_y (red) and N_z (navy) components of the total tidal torque.

As seen from Figure 11 that there is only one pronounced peak in the power spectrum. This occurs because the amplitude of the spectral peak corresponding to the half-period of Jupiter is too large in comparison with others

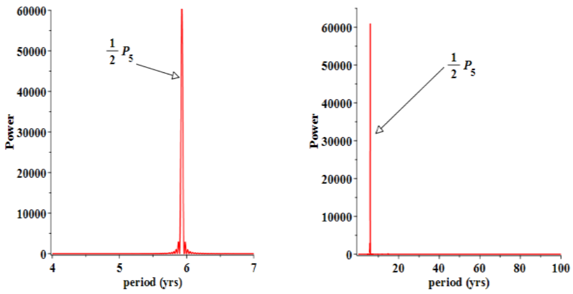


Figure 11 – Power spectra of the total tidal torque.

Now, to obtain a power spectrum, we will first digitize the total torque (we take only N_z – component) with a time step of 0.0025 yr (< 1 day). This step is much lower than the Nyquist critical period, $P/2$, which in this case is equal to $P_1/(2 \cdot 12)$ ($\approx$ 0.01 yr, because of that the minimum period is equal to $P_1/12$, but not to the period of Mercury, P_1, or $P_1/2$, see the formula (6) and Appendix C), and therefore such a sample implementation should not affect the spectrum (for more details, see Section 5, Section 5.1 and Section 5.1.1).

To see other peaks, it is necessary to separately consider the different ranges of the power spectrum, for example, as shown in Figures 12 and 13.

Passing through the power spectrum step by step, it is not difficult to calculate all the periods occurring and compare them with those obtained from analytical calculations, to see how reliable the tool is. Calculations using nonlinear optimization show that the predicted and computed periods are identical (see Appendix D).

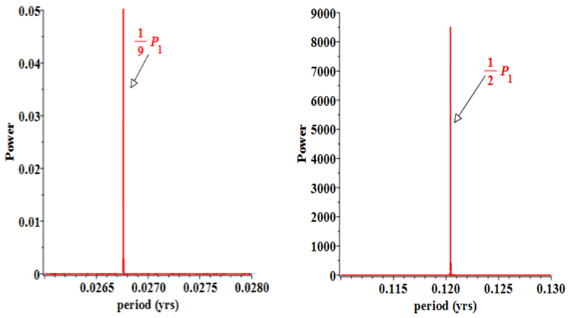


Figure 12 – Some examples of power spectra: the case of Mercury

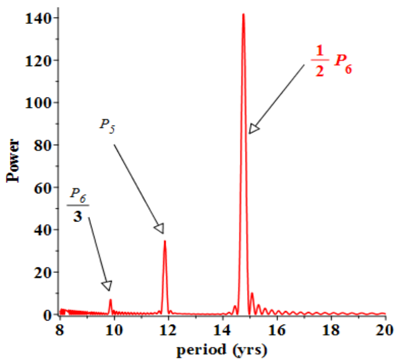


Figure 13 – Some examples of power spectra: the case of Jupiter and Saturn.

Sampling and Averaging

It happens that averaging a dataset over a certain interval and sampling it with the same step is confused. Perfectly performed averaging is the same as smoothing. Oscillations with periods less than the averaging step, during the averaging will be removed, to be precise, will be weakened.

Understanding the intricacies of aliasing during sampling (or averaging) is crucial for the correct detection of the spectrum. Aliasing is effective for periods if the discretization step, δ, (same as the sample period, or sampling interval) exceeds half the maximum period among them (the Nyquist criterion) in a multiperiodic signal. For all other periods exceeding 2δ, aliasing will not be effective if they do not coincide with (or are not close to) spurious periods.

§1. Aliasing. Sampling $t \rightarrow n\delta$, with t, say, being the time, δ the sampling interval, and n the integer, will make two signals, for example, $sin\ (2\pi t/P \pm 2\pi t/\delta)$ and $sin(2\pi t/P)$, where P is the period, indistinguishable:

$$sin\ (2\pi t/P \pm 2\pi t/\delta) \Rightarrow sin\ (2\pi n\delta/P \pm 2\pi n)$$

$$\Rightarrow\ sin\,(2\pi t/P)$$

.

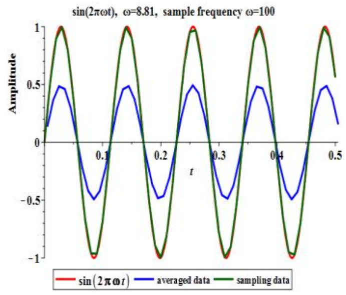
From the above formula it follows that during the sampling of the data, pseudo-periods must be generated. It is accepted to call the appearance of pseudo-periods as aliasing. We have created the routine *SpurPers* (a subprogram in *TiTor*) to automate the search for spurious periods. If P_0 is the signal period, N the number of sampling, and δ the sample period, then all spurious periods can be predicted.

It is interesting to know why the Nyquist sample period, $\delta = P_0/2$, is critical. Using *SpurPers* one finds that in the case of $\delta = P_0/2$, n = 1, the pair of spurious periods are 0.33 P_0 and P_0, in other words, the greatest spurious period coincides with the main one, P_0. After that, when $\delta > P_0/2$, the spectrum begins to be distorted with the appearance of periods higher than P_0. The period interval below P_0 is also filled with spurious periods. It also shows that whenever a sample occurs, aliasing also takes place, regardless of whether the sample period is greater than or less than the critical value. In the second case, they collect below the value of the basic period, and thus it goes beyond the scope of interest of the investigation.

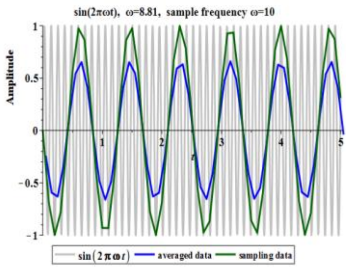
Consider three examples that can teach us the data analysis no less than the theory. First, consider a single sinusoidal function with a period of $P = 1/8.81$ (arbitrary units), $sin(2\pi t/P)$, t being an argument, say time, and three cases of its averaging and sampling with the same steps, $1/100, 1/10, 1$, what is considered by Poluianov and Usoskin [56].

As seen from Figure 14, in all cases of the two curves, both the averaged and sampled data have the same time dependence (in Figure 14 the amplitudes of the averaged data are increased to be better seen). This means that they both will have a similar power spectrum.

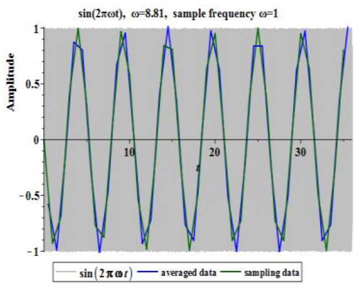
In a forward problem, when P and δ are given, using *SurPers*, it can be predicted what spurious periods will appear in the spectrum, moreover, what period will mainly define the time-dependence of the sampled data. There are many spurious periods, so let's choose some of them which are in our interest (Table 2).



(a)



(b)



(c)

Figure 14 – Averaged (blue) and sampled (green) data for $sin(2\pi 8.81 t)$. Three cases of averaging and sampling over the period of: (a) = 1/100; (b) = 1/10; (c) = 1.

Table 2: Spurious periods for sampled data of $sin(2\pi\omega_0\, t)$, ω_0= 8.81

	n	1	2	3
I: ω_s=100	P	0.011	0.005	0.003
	ω	91.19	191.19	291.19
	n	1	2	3
II: ω_s=10	P	**0.840**	0.0894	0.047
	ω	1.190	11.19	21.19
	n	8	9	10
III: ω_s=1	P	1.235	**5.263**	0.840
	ω	0.810	0.190	1.190

Recall that, P is the spurious period, δ the sample period, ω_s is the sample frequency, δ =$1/\omega_s$, ω the spurious frequency, $\omega = 1/P$. Note that, in the third case (see Table 2), the spectrum of spurious periods covers the spectra of first two cases. The underlined periods, $P = 0.840$ and $P = 5.263$ in Table 2, are the main periods in the sense that they define main ('visible') time dependence in cases II and III, respectively (see Figure 13). Consider separately the power spectra in those three cases for the averaged and sampled data (Figure 14 and Figure 15). Poluianov and Usoskin [56] note that "*The first two signals with $f_s < f_N$ (the authors have in mind $\omega_s = 1$ and $\omega_s = 10$) are distorted and their spectral peaks are shifted from the true position of f_0 to frequencies 0.188 and 1.189. The last one with $f_s > f_N$ (i.e., $\omega_s = 100$) does not have any aliasing distortion. Its spectral peak stands at the frequency that is equal to f_0*" (here f_0, f_s and f_N are the main frequency, the sample and the Nyquist ones, respectively).

No, there are some arguments to object to them:

(a) In the cases of ω_s = 10 and ω_s = 1, the spectra of sampled and averaged data also contain the true frequency, ω_0 = 8.81 and it is not shifted from the true position (Figure 15).

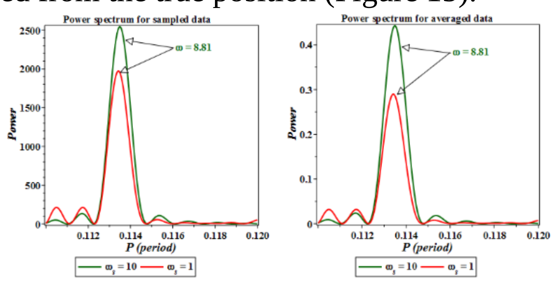


Figure 15 – Power spectra for the sampled (left) and averaged (right) data of $sin(2\pi \cdot 8.81 \cdot t)$: the range of the true spectrum.

In a forward problem to single out it among the noisy environment of spurious periods is an easy task, however it is not easy to do in the case of the inverse problem. The matter is that the spectrum of spurious periods forms a group, i.e., any period from a spurious spectrum generates the same spectrum except itself. Thereby, it does not allow to find the true period without certain additional conditions. The latter is a difficult work.

(b) The frequencies of 0.188 and 1.189 (the precise values of which are 0.19 and 1.19, see Table 2, Case II, n = 1 and Case III, n = 9, 10), which have already been predicted, are indeed spurious, but not the true frequency ω_0 (= 8.81) shifted from the true position (see item (a)).

(c) Spurious frequencies, in the case of ω_s = 100, also arise, but they are simply ignored because they are gathered in the frequency range higher the critical Nyquist frequency $2\omega_0$ (Case I, n = 1, 2, 3).

Figure 16 shows that both the averaged and sampled data of single harmonic function in all three cases have the same spectra. It is important to draw attention to that the power amplitudes for the averaged data are much less than those for the sampled data.

As we shall see, it is crucial in the multiperiodic case when the data are averaged over a certain interval.

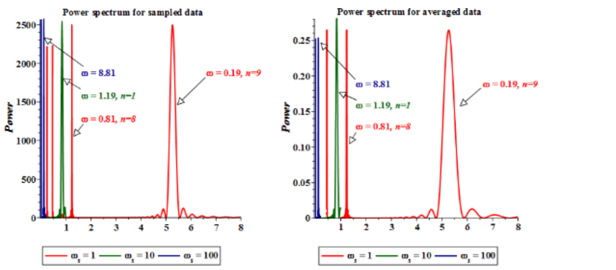


Figure 16 – Power spectra for sampled (left) and averaged (right) data of single sinusoid, $sin(2\pi \cdot 8.81 \cdot t)$.

As a second example, which should describe a multiperiodic case, consider the time series of the total tidal torque, N_z.

§2. Averaging. In the considered case, when the time series of the tidal torque is annually averaged, all the oscillations related to Mercury, Venus and Earth are smoothed out. Except for the period of $P_4 =$ 1.88 years, associated with Mars (and periods lower than $P_5/6$ = 1.98, i.e., P_5/7, ...), all other periods satisfy the condition P_j > 2 years, where j refers to the sequence number of the planets, $j \in$ [5, 6, 7, 8]. Using *SpurPers* we obtain that the greatest spurious periods > 15 years, generated by the periods of Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune, are (for a detail see Appendix F, Table F1):

$SpPr$ = [15.8, 18, 19.7, 23, 25, 42.8, 59, 84.44, 88.4, 109, 119, 130, 138, 145, 148, 163, 186, 217, 218, 261, 266, 326, 435, 652, **1304**] (units in year).

The spurious period of 1304 years, relevant to the Earth (Table F1), is the greatest in a range of long periods, and one can expect that the oscillation with a period of its half of 652 years (remember above discussion concerning power of oscillation of period $P/2$, moreover, it is necessary to pay attention to the fact that the period of 652 years arises as a spurious one separately, see Table F1), will be effective in both cases of data averaging and sampling.

It is a rare occasion when the greatest spurious periods generated by P_j/n, where $n \in$ [1, 2, ..., 12], coincide with P_{sp}/n, where P_{sp} is one of the spurious periods from the spectrum. In the case of the Earth, the spurious periods (we omit the years in the values of the periods) $1304/n$ and the greatest spurious periods generated by P_3/n, coincide (see Table F1). Notice here that the value of a spurious period, for example, of ≈ 1304 years, is not absolute and depends on what orbital period for the Earth (for the planet) is taken and what sample period is accepted. The value of such a period makes sense only to find it as spurious (from a comparison what the analytics gives, using *SpurPers*, with that is found from the power spectrum) and remove it subsequently.

There is another fear connected with suppressing the amplitudes of short-term oscillations (relevant to the tidal torque of Mercury, Venus and the Earth) owing to averaging. Note that the generation of long periods by pairs (sets) of periods is effective when the oscillations are of comparable amplitudes (Appendix E). Averaging can result in the fact that probable long periods generated by superposition of a pair of periods of inner and outer planets will not be effective because the amplitudes become incomparable. Nevertheless, analysis using

Equation (C1) and Table E1 (see Appendix C and Appendix E) shows that there are no such pairs of periods with comparable amplitudes.

Figure 17 illustrates the annually averaged tidal torque for all planets and the non-averaged torque for planets from Mars to Neptune. Both curves coincide, and this shows that during averaging, the oscillations associated with the tidal torques of Mercury, Venus and Earth are almost removed.

Most periods can be calculated from the power spectrum of the averaged data, but we are interested in a range of long periods, and the spectra for this part in two cases of averaging and sampling in increments of one year are strongly differ from each other. Figure 18 shows that in the case of sampling the spurious period of 652 years there is in a spectrum, but it cannot be said about an averaging case. In the latter case the period of 652 years disappears.

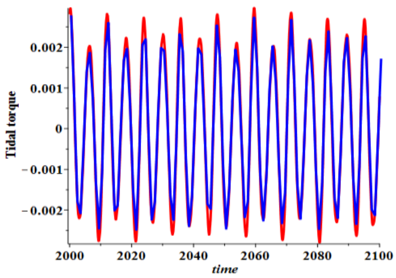


Figure 17 – Tidal torque for planets from Mars to Neptune (red), and annually averaged torque for all planets (blue).

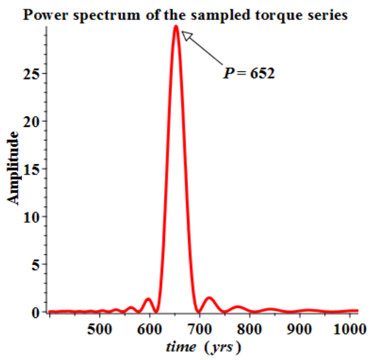


Figure 18 – The spurious period of 652 years in the spectrum of sampled total tidal torque.

The spurious half-period of 652 years is so strong that it suppresses other long periods. The power of the spurious period of 652 years in case of the annually sampled data is more powerful than in case of the averaged data. Therefore, the power spectrum of the averaged data is substantially free of the spurious periods caused by the three first planets.

To confirm the last, we will bring a third example.

§3. Attenuation of the power of spurious periods. Here is another example showing that averaging weakens the power of spurious periods. Let $f\,(t)$ be a simple periodic function with periods $P_1 = 1.99, P_2 = 3.1, P_3 = 11$, and $P_4 = 411$ (arbitrary unit):

$$f\,(t) = \sum_1^4 sin(2\pi t/P_k)\,.$$

Let's sample and average this function over an interval of 2. The length of the dataset is 10000. Using *SpurPers* we find that the greatest spurious periods generated by P_1, P_2, P_3 and P_4 are 398.0, 5.6, 2.4 and 2.0, respectively.

The spurious period 398 generated by P_1, is close to period of 411, and one can expect that it is likely to distort the spectrum.

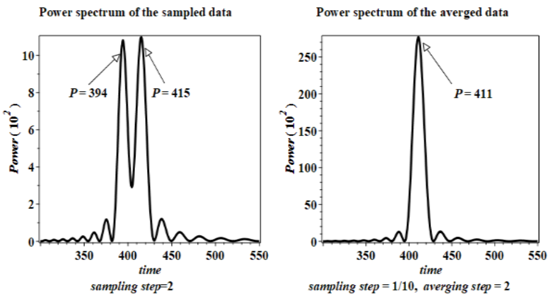


Figure 19 – The spurious period of 398 and the true period of 411 in the spectra of sampled (left) and averaged (right) data.

We see from Figure 19(a), which shows the case of sampling, the periods 398 and 411 are shifted accordingly to 394 and 415, and therefore it is impossible to correctly determine their true values, firstly. Both of them are so powerful that in case of the inverse problem it would be hard to say which of them is the true period and which is spurious one, secondly.

In the case of averaging, Figure 19(b) shows only the true period 411, which is slightly shifted from the true position for 0.3. It is the result of that the oscillation corresponding to the period 398 is only weakened for 400 times during the averaging, but not completely removed. For the sake of rapid calculation during the averaging, we replaced the integration by quadrature with a step $1/10$, so the averaging does not smooth the curve well enough. When the summation step decreases from $1/10$ to $1/250$, the shift will be reduced to 0.07 (instead of 0.3). Thus, we have come to the fact that averaging weakens the power of spurious periods and allows us to correctly find the true period.

Note also, as seen from Figure 19, that in the multiperiodic case, the spectra of sampled and averaged data unlike the case of a single-periodic function (§1 of this section) do not coincide.

FFT

In the *FFT* spectra of the annually averaged and sampled total tidal torque (Figure 20), covering a period interval of $50 - 2000$ years, there is only one peak corresponding to 625 years, which is a spurious period of 652 years, shifted for 27 years.

The length of the dataset is taken to be of 10000 years. Spurious periods of both 326 and 435 years are also found in the spectrum, but they are very weak and also displaced for 4%. Shifting periods is almost the rule for *FFT*. For example, the same displacement of 4% exists also for the exemplary function of $sin(2\,\pi t/5.9297\,) + (1/10)\,sin(2\,\pi t/652)$. Note that the value of shifting depends on the length of the dataset. If to increase the length of the dataset from 1000 to 10000 years, then the shifting will be reduced from 23% to 4%. In addition to this, *FFT* produces spurious periods.

This, together with what has just been said, means that it is impossible to precisely determine the periods from the *FFT* spectrum.

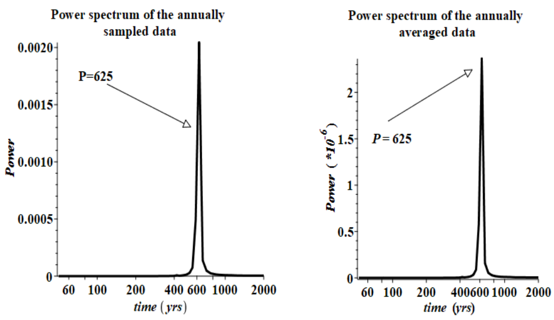


Figure 20 – FFT spectra of the annually averaged (left) and sampled (right) total tidal torque.

Outcome

Thus, we have come to conclusions:

(a) in the forward problem one can predict both the true spectrum and the spurious one for the sampled data;

(b) in the case of a single harmonic function, the power spectra of sampled and averaged data are almost identical;

(c) an example using a single harmonic function to compare the spectra of the averaged and sampled data is inadequate to the multiperiodic case;

(d) in a multiperiodic case, the data averaging in the short-term range does not violate the data analysis, even facilitates the search for long periods and reduces the power of spurious periods when their generators are associated with short-term periods;

(e) *FFT* produces spurious periods and shifts all periods. The latter is particularly important in the long-periodic range. Consequently, the *FFT* spectrum cannot be accepted as a “true spectrum”.

And, at last, we conclude that the argue by Poluianov and Usoskin [56] regarding the spectral analysis conducted by Abreu et al. [1], is based on incorrect prerequisites and cannot be accepted as well-grounded.

Discussion

Some time ago T. Zaqarashvili, knowing about the program-package *SolAct*, has asked our opinion about the paper of Abreu et al., since that paper has been subjected to many criticisms. We have decided to express our opinion not only about this paper, but also about the critical remarks which are devoted to this paper and to the planetary hypotheses in general. I would like to thank T. Zaqarashvili for the discussion.

At times, some paper plays an important role in the development of some ideas. We believe that in the case of J. A. Abreu, J. Beer, A. Ferriz-Mas, K. G. McCracken and F. Steinhilber [1] it just happened. However, it is necessary to give due regard to criticism, whose role in the development of science is incontestable. At the same time, we must not forget what Samuel Richardson said: *It is much easier to* *find fault with others, than to be faultless ourselves.*

So far the influence of the planets on the Sun was only a hypothesis. Certain mechanisms for the spin-orbit interaction of the Sun and planets have now been suggested. It is only the first step in transformation of the hypothesis into a theory.

Historically, Copernicus' revolution created a new style of thinking that recognizes the sun as the sole ruler of the solar system. Wolf tried to restore the 'rights' of the planets for control in the solar system, drawing attention to their possible impact on solar activity. These attempts are still going on, and it becomes clear that the planets 'have a claim' to take part in the control of both mechanics and the physics of the Sun.

It is necessary to esteem the Wolf vision that throws light along centuries on the understanding of the problem of solar activity.

APPENDICES

APPENDIX A: MODULE SOLACT

Program-package called *SolAct* (**Sol**ar **Ac**tivity) is created to simulate 3d motions of the Sun and planets around the center of mass of the solar system. *SolAct* generates coordinates of the Sun and planets in both the Sun and barycenter centered solar systems, calculates velocities of the planets, angular momenta and other parameters and solves the system of equations that govern the spin-orbit interaction.

1. The module *SolAct* exports many *routines*, which

- draw a trajectory of the planets around the Sun;
- animate the motion of planets about the Sun;
- animate the solar center motion around the barycenter;
- animate the vector from the barycenter to the solar center, the tangent vector, the years of solar activity;
- solve the system of equations of the spin-orbit coupling;
- depict the course of the solar angular velocity and the Wolf numbers with time.

2. The module *SolAct* allows to calculate the correlation coefficients of the Sun's spin with Wolf numbers, curvature and torsion of the position vector of the Sun's center relative to the barycenter and a large number of other parameters.

APPENDIX B: MODULE TITOR

The program-package *TiTor* (Module) is created to calculate the tidal torque. It uses some Routines from *SolAct* (Appendix A) and *DaImAn* (**Da**ta **Im**port and **An**alysis, Aliyev [8]).

The module *TiTor*,

- calculates planetary tidal torques with and without the solar rotation;
- graphically illustrates the tidal torque for both a single planet and all together;
- illustrates the power spectrum, allows the correct calculation of extrema using nonlinear optimization;
- samples continuous functions and averages data for a given step;
- has subroutines for analytically searching for both long (generated by a pair of periods) and spurious periods (in a forward problem).

APPENDIX C: APPROXIMATE FORMULA FOR TIDAL TORQUE

In the formula of tidal torque (N_z component for the ith planet),

$$Nz,i \propto m_i \frac{x_i y_i}{|r_i|^5},$$

$\frac{x_i y_i}{|r_i|^5}$ can be expressed as,

$$\frac{x_i y_i}{|r_i|^5} \propto \frac{(cos\,(i) - \varepsilon_i)\sqrt{1-\varepsilon_i^2}\,sin\,(\xi_i)}{a_i^3(1-\varepsilon_i \cos(\xi_i))^5}.$$

Substituting $\xi_i \approx \omega_i t$ ($\xi_i = \omega_i t + \varepsilon_i \sin(\xi_i)$) into this formula, after some simplification we arrive at Equation (C1) (omitting the factor $1/2048 \cdot \sqrt{1-\varepsilon_i^2/a_i^3}$,

$$\frac{x_i y_i}{|r_i|^5} \propto -1024\,sin\,(\omega_i t) + (-512\,sin\,(\omega_i t - 2560\,sin\,(3\omega_i t))\,\varepsilon_i + (-2560\,sin\,(2\omega_i t) - 3840\,sin\,(4\omega_i t))\,\varepsilon_i^2 + (-3840\,sin\,(3\omega_i t) - 3840\,sin\,(5\omega_i t))\varepsilon_i^3 + (960\,sin\,(2\omega_i t) - 3840\,sin\,(4\omega_i t) - 2880\,sin\,(6\omega_i t))\varepsilon_i^4 + \big(3360\,sin\,(\omega_i t) + 2592\,sin\,(3\omega_i t) - 2400\,sin\,(5\omega_i t) - 1632\,sin\,(7\omega_i t)\big)\varepsilon_i^5 + \big(6240\,sin\,(2\omega_i t) + 2976\,sin\,(4\omega_i t) - 1056\,sin\,(6\omega_i t) - 720\,sin\,(\omega_i t)\big)\varepsilon_i^6 + \big(3840\,sin\,(\omega i t) + 6240\,sin\,(3\omega i t) + 2400\,sin\,(5\omega_i t) - 240\,sin\,(7\omega_i t) - 240\,sin\,(9\omega_i t)\big)\varepsilon_i^7 + \big(4200\,sin\,(2\omega_i t) + 3840\,sin\,(4\omega_i t) +$$

$1260\, sin\,(6\omega_i t) - 60\, sin\,(10\omega_i t))\varepsilon_i^8 +$
$(1260\, sin\,(\omega_i t) + 2460\, sin\,(3\omega_i t) +$
$1650\, sin\,(5\omega_i t) + 490\, sin\,(7\omega_i t) +$
$30\, sin\,(9\omega_i t) - 10\, sin\,(11\omega_i t))\varepsilon_i^9 +$
$(708\, sin\,(2\omega_i t) + 795\, sin\,(4\omega_i t) +$
$430\, sin\,(6\omega_i t) + 116\, sin\,(8\omega_i t) +$
$10\, sin\,(10\omega_i t) - sin\,(12\omega_i t))\varepsilon_i^{10} +$
$(84\, sin\,(\omega i t) + 180\, sin\,(3\omega_i t) +$
$150\, sin\,(5\omega_i t) + 70\, sin\,(7\omega_i t) +$
$18\, sin\,(9\omega_i t) + 2\, sin\,(11\omega_i t))\varepsilon_i^{11}.$
(C1)

where a_i is the same as it was designated in Section 4, ε_i and ω_i are the eccentricity and circular frequency of the ith planet, respectively. As can be seen from this expression, the terms with the frequency $2\omega_i$ ($\omega_i = 2\pi/P_i$), or with the period $P_i/2$, will have the greatest amplitude (because of eccentricities are of small values).

APPENDIX D: PREDICTED AND COMPUTED PERIODS

Using *TiTor*, it can be calculated all the periods from the power spectrum of the total tidal torque series, some of which are shown below. Calculations using nonlinear optimization show that the predicted and computed periods are almost identical:

Table D1: Comparison of the given and found periods from spectrum

	Mercury	***Venus***	***Earth***
***P*/2 (years)**	0.120426	0.307603	0.499616
Found	0.120426	0.307603	0.499974
***P* (years)**	0.240852	0.615206	0.999233
Found	0.240852	0.615188	0.999181
	Mars	***Jupiter***	***Saturn***
***P*/2 (years)**	0.940480	5.92980	14.7502
Found	0.940499	5.92975	14.7610
***P* (years)**	1.88096	11.8596	
Found	1.89396	11.8605	

APPENDIX E: THE EMERGENCE OF LONG-TERM OSCILLATIONS

A pair of oscillations with close periods generates a long-term oscillation. For example, two sinusoids with close periods of 10 and 10.8 years generate a long-term oscillation with a period of 135 years, which can be easily seen from Figure E1.

Are there pairs and triples (also, doublets and triplets connected with the period of the same planet) with close periods in tidal torque? Yes, there are too many various combinations of pairs of periods. Therefore, we automated the search and calculation of long periods, some of which are shown in Table E1.

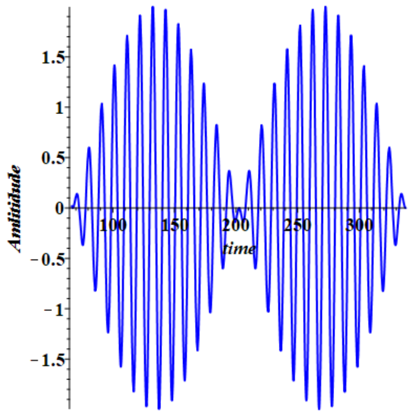


Figure E1 – Addition of two sinusoids with close periods.

However, if the difference between the amplitudes of oscillations with close periods is very large, the appearance of long-term periodic modulation will be ineffective.

Table E1: Long periods generated by pairs of close periods[1]

pairs	P_{long}	***pairs***	P_{long}
$P_6/8$, $P_5/3$	55	$P_7/10$, $P_6/3$	61
$P_7/8$, $P_7/7$	85	$P_7/8$, P_5	98
$P_6/2$, $P_8/10$	132	$P_8/10$, $P_8/9$	166
$P_6/4$, $P_7/11$	179	$P_7/5$, $P_8/9$	203
$P_7/9$, $P_6/3$	214	$P_7/6$, $P_6/2$	321
$P_8/6$, P_6	450	$P_6/10$, $P_5/4$	589
P_5, $P_7/7$	629	$P_8/10$, $P_7/5$	906
$P_6/5$, $P_5/2$	1177	$P_8/6$, $P_7/3$	1510
$P_8/4$, $P_7/2$	2265	$P_8/2$, P_7	4530

[1]unit of periods is year

APPENDIX F: SPURIOUS PERIODS GENERATED BY PLANETS

The subroutine *SpurPers* allows to find spurious periods. Below it is shown only a part of the spurious periods (> 15 years) generated by the periods of Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune:

Table F1: The greatest spurious periods[1]

	P	P/2	P/3	P/4	P/5	P/6	P/7
Venus		266					
Earth	1304	652	435	326	261	217	186
Mars					15.8		
Jupiter	84.44						
Saturn		59		19.7	59		19.7
Uranus					218	138	

[1]unit is year

In Table F1 the first row shows the periods of the planets and the pair [planet, period] defines the source of the spurious period. For example, [Venus, $P/2$] means that a spurious period of 266 years (see Table F1) is generated by the half-period of Venus.

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Accepted to print at 13.01.2021