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Yu.V. Arkhipov [1], A. Askaruly [1], A.B. Ashikbayeva [1],
D. Yu. Dubovtsev [1], S.A. Syzganbayeva[1], I.M. Tkachenko[2]
[1]Al-Farabi Kazakh National University, IETP, Almaty, Kazakhstan;
[2]UniversitatPolitècnica de València, Valencia, Spain

DYNAMICAL PROPERTIES OF DENSE ONE-COMPONENT PLASMAS WITHIN THE METHOD OF MOMENTS INTERPOLATION MODEL

Abstract. In this paper we present the results of investigation of dynamic characteristics of dense one-component plasmas. In order to determine the dynamic structure factors (DSF) of such systems, the interpolation self-consistent method of moments is employed, within which the frequency moments are calculated as interpolations of the asymptotic relations for the exact moments. The advantage of this method is the possibility of using approved fitting ratios for internal energy, which speeds up the computations. The obtained results, namely, the position, the height of the DSF are in a good agreement with the computer modelling data in a broad range of variation of the system parameters. Therefore, the conclusion is made that the interpolation self-consistent moment method can be used to calculate dynamic structural factors and other dynamic characteristics of a dense nonideal plasma.

Knowledge of DSF, in particular, allows calculating the dispersion of plasma waves, their dissipation, as well as diagnosing the parameters of the medium under consideration.

Keywords: dense plasma, method of moments, dynamic properties

Introduction

Dense one-component plasmas are good model systems possessing basic properties of the working bodies of future inertial-fusion devices. Contemporary methods of diagnostics of such systems require knowledge of dynamic characteristics of these Coulomb, though classical, systems.

Recent computational developments [1] permit to describe their static correlations in a quite reliable way. This cannot be said about the dynamic properties. On the other hand, the self-consistent approach based on the non-canonical solutions of the Hamburger moment problem [2] facilitates a direct relation between the dynamic and static characteristics of the above non-perturbative systems under the extreme inertial-fusion conditions.

Here, the results of the simplified interpolation version of the moment approach with respect to the OCP dynamic properties are compared to the simulation data of [3]. Thus, the robustness of the self-consistent method based on sum rules and other exact relations, see [4, 5] and references therein, is demonstrated.

The interpolation self-consistent method of moments.

Consider the classical one-component system, which is characterized by a coupling parameter:

$$\Gamma = \frac{\beta (Ze)^2}{a}, \qquad (1)$$

Here β^{-1} stands for the temperature in energy units, Ze designates the ion charge, and$a = (3/4\pi n)^{1/3}$ is the Wigner-Seitz radius with n being the number density of charged particles.

Plasma is considered to be strongly-coupled if $\Gamma > 1$ and is ideal for $\Gamma \ll 1$. The upper limit of the coupling parameter is determined by the Wigner crystallization.

The interparticle interaction is described by the Coulomb potential:

$$\varphi(r) = \frac{(Ze)^2}{r}, \qquad (2)$$

The central characteristic of dynamic properties of plasmas is the dynamic structure factor (DSF)$S(k,\omega)$, a positive frequency function, which is associated with the dielectric function$\epsilon(k,\omega)$ (via the classical version of the fluctuation-dissipation theorem):

$$-\frac{Im\,\epsilon^{-1}(k,\omega)}{\omega} = \frac{(2\pi Ze)^2\beta}{k^2} S(k,\omega). \qquad (4)$$

Here, the wavenumber k (the system under consideration is presumed to be uniform) is a parameter.

Consider five convergent sum rules which are frequency power moments of the system DSF,

$$S_v(k) = \frac{1}{n}\int_{-\infty}^{\infty} \omega^v S(k,\omega)d\omega,$$
$$v = 0,1,2,3,4. \qquad (5)$$

All odd-order moments vanish since the DSF is an even function of frequency in a statistically classical system.

It is well known [6] that the analytic prolongation of the positive function of frequency, DSF, onto the complex-frequency upper half-plane $Im\ > 0$is constructed by means of the Cauchy integral formula,

$$S(k,z) = \frac{1}{n}\int_{-\infty}^{\infty} \frac{S(k,\omega)}{\omega - z}d\omega; \quad (6)$$

it admits the asymptotic expansion:

$$S(k,z) \cong -\frac{S_0(k)}{z} - \frac{S_2(k)}{z^2} - \frac{S_4(k)}{zz^5} - -0\left(\frac{1}{z^5}\right), \ Im\ \geq 0. \quad (7)$$

The zero-order moment is, obviously, the static structure factor SSF, $S_0(k) = S(k)$, while the second moment is equivalent to the f-sum rule,

$$S_2(k) = \omega_p^2\left(\frac{k^2}{k_D^2}\right) = \omega_p^2\left(\frac{q^2}{3\Gamma}\right), \qquad (8)$$

and the fourth moment equals

$$S_4(k) = \omega_p^2 S_2(k)\left(1 + \frac{3k^2}{k_D^2} + U(k)\right),$$
$$= \omega_p^2 S_2(q)\left(1 + \frac{q^2}{\Gamma} + U(q)\right). \quad (9)$$

Here $q = ka$, $\omega_p = \sqrt{4\pi n(Ze)^2/m}$ refers to the plasma frequency, and $k_D = \sqrt{4\pi n(Ze)^2\beta}$ is the Debye wavelength with m being the mass of the particles.

The contribution $U(k)$ in (9) is directly related to the interaction between particles. In 1998, the authors of [7] obtained the interpolation expression for this contribution based on the hypernetted-chain approximation:

$$U(k) = -\frac{4\,k^2}{15}\frac{\Gamma^{\frac{3}{2}}}{\beta\,\mathrm{m}}\left(\frac{-0.905}{\sqrt[4]{0.632 + \Gamma}}{}^{2}_{3} + . + \frac{0.272}{1+\Gamma}\right) \qquad (10)$$

Collecting all above expressions together, the characteristic frequencies [4], needed in the following, are written in the following form:

$$\omega_1^2 = \omega_1^2(k) = = \omega_p^2\left(1 + k^2k_D^{-2} + k^4k_q^{-4}\right), (11)$$

$$\omega_2^2 = \omega_2^2(k) = \omega_p^2\Big(1 + \frac{3\,F_{\frac{3}{2}}(\eta)}{\omega_p^2 m\,\beta\,\theta^{-3/2}}k^2 + \left(\frac{\hbar}{2m}\right)^2\frac{k^4}{\omega_p^2} - -\frac{4\,k^2}{\omega_p^2}\frac{\Gamma^{\frac{3}{2}}}{15\beta\,\mathrm{m}}\Big(\frac{-0.905}{\sqrt[4]{0.632 + \Gamma}}{}^{2}_{3} + \frac{0.272}{1+\Gamma}\Big)\Big). (12)$$

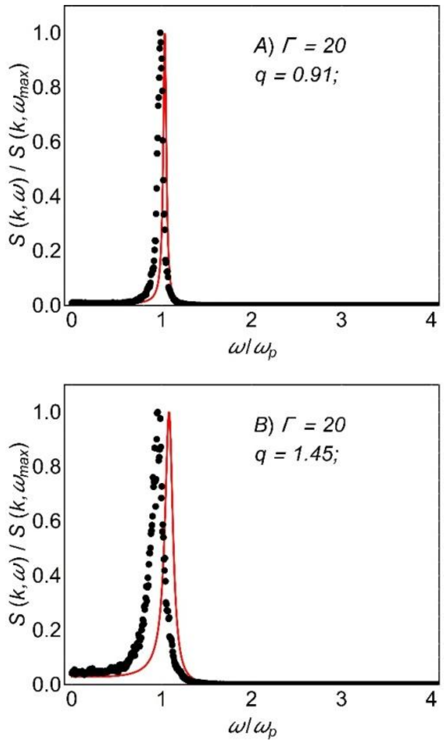
Thecharacteristic frequencies determine the plasma inverse dielectric function(IDF) and, finally, the system dynamic characteristics. Indeed, using Nevanlinna's formula [8] one obtains for the IDF the linear-fractional transformation of the function of the Nevanlinna class $Q(k,z)$:

$$\epsilon^{-1}(k,z) = 1 + \frac{\omega_p^2(Q(k,z) + z)}{z\left(z^2 - \omega_2^2(k)\right) + Q(k,z)(z^2 - \omega_1^2(k))}. (8)$$

Both the IDF and the Nevanlinna parameter function (NPF) $Q(k,z)$ are analytic in the upper half-plane $\mathrm{Im}z > 0$and havethere positive imaginary parts, and, in addition, the NPF satisfies the limiting condition $\lim_{z\to\infty}\frac{Q(k,z)}{z} = 0$ which guarantees the fulfillment of the moment conditions (5) for any correct NPFD. Note that the Nevanlinna function $Q(k,z)$is modelled here, like in [8], in the static approximation with $Q(k,z) = Q(k,0) = ih(k),\ \ h(k) > 0.$

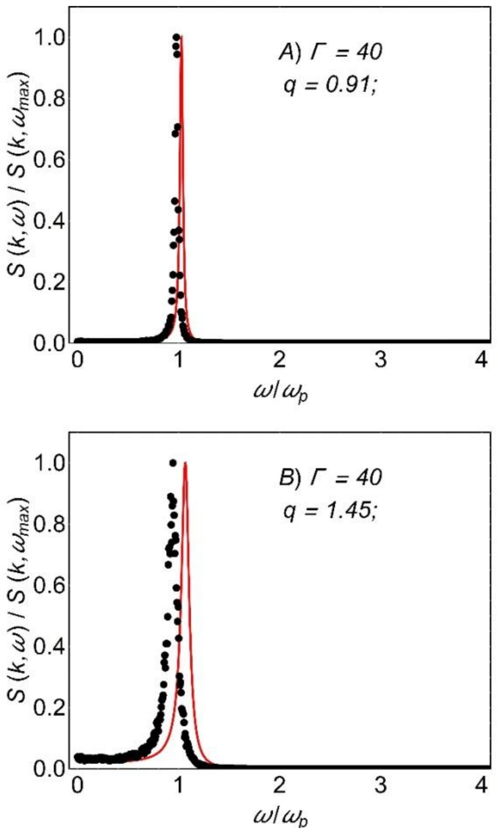
Results.

The numerical results on the dynamic structure factor are compared to the simulation data of [3] and are summarized in Figures 1-2. In all figures the squares correspond to the data of [3] with $q = ka$ being the dimensionless wavenumber.



A) *q=0.91*andB) *q = 1.45.*

Figure 1 – The OCP normalized dynamic structure factor (red lines) in comparison with the simulation data of [3] (points) at Γ=20



A) *q=0.91*andB) *q = 1.45.*

Figure 2 – The OCP normalized dynamic Structure factor (red lines) in comparison with the simulation dataof [3] (points) atΓ=40

Conclusions

The reliability and robustness of the self-consistent moment approach is demonstrated by comparing the results obtained for the OCP dynamic structure factors with the simulation data.No adjustment to the latter is employed Using the obtained dynamic structural factors, one can determine both the dispersion and attenuation of plasma waves in the plasma under consideration. Such an outcome can be used in dense plasma diagnostics.

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Ю.В. Архипов [1], Ә. Асқарұлы[1], Ә.Б. Ашықбаева[1],
Д.Ю. Дубовцев [1], С.А. Сызғанбаева [1], И.М. Ткаченко [2]
[1]*Аль-Фараби атындағы ҚазҰУ, Алматы, Қазақстан;*
[2]*Валенсия политехникалық университеті, Испания*

ТЫҒЫЗ БІР КОМПОНЕТТІ ПЛАЗМАНЫҢМОМЕНТ ӘДІСІНІҢ ИНТЕРПОЛЯЦИЯЛЫҚ МОДЕЛІ ШЕҢБЕРІНДЕ ҚАРАСТЫРЫЛҒАН ДИНАМИКАЛЫҚ ҚАСИЕТТЕРІ

Аннотация. Осы жұмыста тығыз біз бір компонентті плазманың динамикалық сипаттамаларын зерттеу нәтижелерін ұсынамыз. Мұндай жүйелердің динамикалық құрылымдық факторларын (ДҚФ) анықтау үшін интерполяциондық өзіндік келісілген моменттер әдісі қолданылады, оның шеңберінде жиіліктік моменттер нақты моменттер үшін асимптотикалық арақатынастың интерполяциясы ретінде есептеледі. Бұл әдістің артықшылығы ішкі энергия үшін бекітілген тарап арақатынасын пайдалану мүмкіндігі болып табылады, бұл есептеулерді тездетеді. Алынған нәтижелер, атап айтқанда, ДҚФ биіктігі жүйе параметрлерінің кең диапазонында компьютерлік модельдеу деректерімен жақсы үйлеседі. Сондықтан интерполяциялық өзін-өзі келісілген жедел әдіс динамикалық құрылымдық факторларды және тығыз емес плазманың басқа да динамикалық сипаттамаларын есептеу үшін пайдаланылуы мүмкін деген қорытынды жасалды.

ДҚФ білу, атап айтқанда, плазмалық толқындардың дисперсиясын, олардың диссипациясын есептеуге, сондай-ақ қарастырылатын ортаның параметрлерін диагностикалауға мүмкіндік береді.

Түйін сөздер: тығыз плазма, момент әдісі, динамикалық қасиеттері.

Ю.В. Архипов [1], Ә. Асқарұлы[1], Ә.Б. Ашықбаева[1],
Д.Ю. Дубовцев [1], С.А. Сызғанбаева [1], И.М. Ткаченко [2]
[1]*КазНУ им. аль-Фараби, Алматы-Казахстан*;
[2]*Валенсийский политехнический университет, Испания*

ДИНАМИЧЕСКИЕ СВОЙСТВА ПЛОТНОЙ ОДНОКОМПОНЕНТНОЙ ПЛАЗМЫ В РАМКАХ ИНТЕРПОЛЯЦИОННОЙ МОДЕЛИ МЕТОДА МОМЕНТОВ

Аннотация. В данной работе представлены результаты исследования динамических характеристик плотной однокомпонентной плазмы. Для определения динамических структурных факторов (ДСФ) таких систем используется интерполяционный самосогласованный метод моментов, в рамках которого частотные моменты вычисляются как интерполяции асимптотических соотношений для точных моментов. Преимуществом данного метода является возможность использования утвержденных сторона соотношения для внутренней энергии, что ускоряет вычисления. Полученные результаты, а именно положение, высота ДСФ хорошо согласуются с данными компьютерного моделирования в широком диапазоне изменения параметров системы. Поэтому сделан вывод, что интерполяционный самосогласованный моментный метод может быть использован для расчета динамических структурных факторов и других динамических характеристик плотной неидеальной плазмы.

Знание DSF, в частности, позволяет рассчитывать дисперсию плазменных волн, их диссипацию, а также диагностировать параметры рассматриваемой среды.

Ключевые слова: плотная плазма, метод моментов, динамические свойства.

Yu.V. Arkhipov [1], A. Askaruly [1], A.B. Ashikbayeva [1],
D. Yu. Dubovtsev [1], S.A. Syzganbayeva[1], I.M. Tkachenko[2]
[1]Al-Farabi Kazakh National University, IETP, Almaty, Kazakhstan;
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